Statistics 583, Problem Set 1

Wellner; 4/1/98

Reading: Chapter 6, sections 6.1 and 6.2; Ferguson, MS, Chapter 5, sections 5.1, 5.2, and 5.5.
Due: Wednesday, April 8, 1998

1. Tell me the tentative topic for your talk and paper. [Please talk with me about this during office hours if you have questions.]

2. Consider the Locally Most Powerful test ϕ for testing $H : \theta \le 0 \equiv \theta_0$ versus $K : \theta > 0 = \theta_0$ in Example 6.1.8.
A. Suggest two different approximations to the power of this test, one for local alternatives (of the form $\theta_n = t/\sqrt{n}$ with $t > 0$), and the other for fixed alternatives, $\theta > 0$.
B. What is the behavior of each of these two approximations for large values of θ? Which of them shows that the power function decreases to 0 as $\theta \to \infty$? Why?

3. Ferguson, MS, problem 2, page 241, parts (a) and (b)

4. A random variable X takes on the values 1, 2, 3, 4 with probability distribution $p_0(x)$ or $p_1(x)$.

x	1	2	3	4
$p_0(x)$	.4	.2	.1	.3
$p_1(x)$	.3	.3	.3	.1

A. Find a most powerful test of size $\alpha = .1$ for testing p_0 versus p_1 and determine its power.
B. Find a test ϕ which minimizes the sum of risks $a + b$ where $a \equiv E_0\phi$ and $b \equiv E_1(1-\phi)$.

5. For P_0 and P_1 as given in problem 4 compute $d_{TV}(P_0, P_1)$, $H(P_0, P_1)$, and the affinity $\rho(P_0, P_1) = \int \sqrt{p_0(x)p_1(x)}d\mu(x)$; for the product laws P_{0n} and P_{1n} (corresponding to observation of $X_1, \ldots, X_n$ i.i.d. P_0 or P_1 respectively) compute $\rho(P_{0n}, P_{1n})$ and $H(P_{0n}, P_{1n})$ for $n = 10, 25, 100$. What does this imply about the test, ϕ_n say, based on $X_1, \ldots, X_n$ which minimizes the sum of risks?

6. **Optional bonus problem:** Ferguson, MS, problem 2, page 241, part (c).