5/16/97 STATISTICS 583, Problem Set 7 **Wellner**

Due: Wednesday, May 20, 1997

Reading: Lecture Notes, Chapter 8, sections 1 and 2.
Efron and Tibshirani, *Bootstrap*, Chapters 1 - 6, Chapter 11.

1. Let X be a positive random variable with d.f. F on R^+, and suppose that

$$\mu_F = EX = \int_0^\infty x dF(x) = \int_0^1 F^{-1}(s)\,ds < \infty .$$

For $0 \le t \le 1$ define

$$L_t(F) = \frac{\int_0^t F^{-1}(s)\,ds}{\int_0^1 F^{-1}(s)\,ds};$$

$\{L_t(F) : 0 \le t \le 1\}$ is the *Lorenz curve*, and is often used in economics to describe inequities in income distributions.

A. Compute and plot the Lorenz curve $L_t(F)$ when:

(i) F is the exponential(λ) distribution, $F(x) = 1 - e^{-\lambda x}$.

(ii) F is the distribution of a constant random variable, $F(x) = 1_{[x_0,\infty)}(x)$.

B. For a fixed $0 < t < 1$, is $L_t(F)$ a weakly continuous functional of F? Explain why or why not.

C. Calculate the Gateaux derivative $\dot{L}_t(F; G - F)$ for a fixed t and find the influence function $\psi_F(x)$ of $L_t(F)$.

D. Discuss approaches to proving asymptotic normality of $L_t(IF_n)$ based on functional derivatives stronger than Gateaux. Which of these approaches is most likely to succeed?

2. The expression for the jackknife variance estimator for the median, (2.2) on page 10 in chapter 8 was derived under the assumption $n = 2m$ and that $T(IF_n) = X_{(m)}$ if $n = 2m - 1$, $T(IF_n) = (X_{(m)} + X_{(m+1)})/2$ if $n = 2m$.

A. Derive the first equality in (2.2), page 10, using this definition of the sample median.

B. Derive versions of (2.2) using $T(F) = F^{-1}(1/2)$ (strictly). Does the asymptotic result in (2.2) still hold?