

Statistics 583, Problem Set 9

Wellner; 5/27/2009

Reading: Wasserman, Chapters 4-9, pages 43-223

Due: Wednesday, June 3, 2009

Reminder: Second mid-term exam: Friday, June 5, 10:30-11:20 AM

- Verify the variance part of Theorem 6.9, Wasserman, page 129.
 - Verify the proof of (6.12) - (6.14) in Theorem 6.11, Wasserman, page 129.
A question about the proof: Is the $o(1/n)$ term on page 142, line -8, correct? Does this have any effect on (6.12)?
 - Can you find the limiting distribution of $n^{1/3}(\hat{f}_n(x) - f(x))$ under hypotheses similar to those used in (a corrected version of) Theorem 6.11?
- Consider the kernel density estimator defined in (6.26), Wasserman, page 132. Show that if the density f and the kernel k satisfy the hypotheses of Wasserman's theorem 6.28, page 133, and $h = h_n$ satisfies the hypotheses of Theorem 6.27, then for fixed $x \in \mathbb{R}$,

$$\sqrt{nh_n}(\hat{f}_n(x) - E\hat{f}_n(x)) \rightarrow_d N\left(0, f(x) \int k^2(x)dx\right).$$

- Under what restriction on h_n does it follow (from (a) together with further analysis of the bias) that

$$\sqrt{nh_n}(\hat{f}_n(x) - f(x)) \rightarrow_d N\left(0, f(x) \int k^2(x)dx\right)?$$

- If $h_n = cn^{-1/5}$ and the hypotheses of (a) hold, find the limiting distribution of $\sqrt{nh_n}(\hat{f}_n(x) - f(x))$.
- Under the same assumptions as in (c), find the limiting distribution of $\sqrt{nh_n}(\sqrt{\hat{f}_n(x)} - \sqrt{f(x)})$.
- Suppose that $x, y \in \mathbb{R}$ with $x < y$. Find the joint limiting distribution of $(\sqrt{nh_n}(\hat{f}_n(x) - f(x)), \sqrt{nh_n}(\hat{f}_n(y) - f(y)))$ under the assumptions in (b) and (c).

- Wasserman, problem 6.9.3, page 143.
 - Show that (6.35) on page 136 holds.
- Bonus optional problem.** In the context of the histogram estimator in problem 1 above, is there a choice of the bin width $h = 1/m$ that leads to the estimator being asymptotically Poisson rather than asymptotically Gaussian?

5. **Bonus optional problem.** (a) Wasserman, problem 6.9.3, page 143.
(b) Verify the formula (6.16) of Wasserman, page 129.