

## Statistics 583, Problem Set 7

Wellner; 5/13/2009

**Reading:** Chapter 8, sections 8.4 - 8.5, pages 18 - 27;

Wasserman, Chapter 3, pages 13-41; begin reading Chapter 4

**Due:** Wednesday, May 20, 2009

1. The expression for the jackknife variance estimator for the median, in the display (1) on page 11 (3rd line from the bottom) in chapter 8 was derived under the assumption  $n = 2m$  and that  $T(\mathbb{F}_n) = X_{(m)}$  if  $n = 2m - 1$ ,  $T(\mathbb{F}_n) = (X_{(m)} + X_{(m+1)})/2$  if  $n = 2m$ .
  - (a) Derive the first equality in (1), page 11, using this definition of the sample median.
  - (b) Derive versions of the development in (1), page 11, using  $T(F) = F^{-1}(1/2)$  (strictly). Does the asymptotic result in (1) still hold? Here is some further explanation of what I mean by “strictly” here: let  $T_1(\mathbb{F}_n) = X_m$  if  $n = 2m - 1$ ,  $T_1(\mathbb{F}_n) = (X_{(m)} + X_{(m+1)})/2$  if  $n = 2m$ . This is one common definition of the median, and this is the definition used in (a). Let  $T_2(\mathbb{F}_n) = \mathbb{F}_n^{-1}(1/2)$ . This is my favorite definition of the median. Note that  $T_2(\mathbb{F}_n) = T_1(\mathbb{F}_n)$  if  $n = 2m - 1$ , but  $T_2(\mathbb{F}_n) \neq T_1(\mathbb{F}_n)$  if  $n = 2m$ . (What is the value of  $T_2(\mathbb{F}_n)$  in this case?)  $T_2$  is the definition of the median to be considered in 2(b)!
2.
  - (a) Wasserman, problem 3.8.3, page 39, modified. Show that the claimed expression for  $v_{boot}$  given in the display for this problem is incorrect and find the correct expression. Here  $v_{boot} = \text{Var}_{\mathbb{F}_n}(T_n)$  where  $T_n = \overline{X}_n^2$ . [Hint: see Dodd and Korn, *The American Statistician* **61** (2007), 127 - 131, and especially their appendix B, pages 130-131. Apparently the formula given by Wasserman in his problem is from Shao and Tu (1995), page 10; as noted by Dodd and Korn, the expression in Shao and Tu is incorrect.]
  - (b) Explain how the resulting formulas relate to how you would estimate the variance of  $\overline{X}_n^2$  via the delta method.
3. Wasserman, problem 3.8.7, page 40.
4. Wasserman, problem 3.8.11, page 41.
5. **Optional bonus problem:** On page 12, line 4 of Chapter 8 of the lecture notes, it is claimed that if  $E_F|X|^r < \infty$  for some  $r > 0$ , then for the median function

$T(F) = F^{-1}(1/2)$  we have

$$n\text{Var}_F(T(\mathbb{F}_n)) \rightarrow \frac{1/4}{f^2(F^{-1}(1/2))}.$$

Prove (or disprove) this claim.