

Statistics 583, Problem Set 7

Wellner; 5/9/2007

Reading: Chapter 8, sections 8.4 - 8.5, pages 18 - 27;

Wasserman, Chapter 3, pages 13-41; begin reading Chapter 4

Due: Wednesday, May 16, 2006

1. The expression for the jackknife variance estimator for the median, in the display (1) on page 11 (3rd line from the bottom) in chapter 8 was derived under the assumption $n = 2m$ and that $T(\mathbb{F}_n) = X_{(m)}$ if $n = 2m - 1$, $T(\mathbb{F}_n) = (X_{(m)} + X_{(m+1)})/2$ if $n = 2m$.
 - (a) Derive the first equality in (1), page 11, using this definition of the sample median.
 - (b) Derive versions of the development in (1), page 11, using $T(F) = F^{-1}(1/2)$ (strictly). Does the asymptotic result in (1) still hold? Here is some further explanation of what I mean by “strictly” here: let $T_1(\mathbb{F}_n) = X_m$ if $n = 2m - 1$, $T_1(\mathbb{F}_n) = (X_{(m)} + X_{(m+1)})/2$ if $n = 2m$. This is one common definition of the median, and this is the definition used in (a). Let $T_2(\mathbb{F}_n) = F_n^{-1}(1/2)$. This is my favorite definition of the median. Note that $T_2(\mathbb{F}_n) = T_1(\mathbb{F}_n)$ if $n = 2m - 1$, but $T_2(\mathbb{F}_n) \neq T_1(\mathbb{F}_n)$ if $n = 2m$. (What is the value of $T_2(\mathbb{F}_n)$ in this case?) T_2 is the definition of the median to be considered in 2(b)!
2.
 - (a) Wasserman, problem 3.8.3, page 39, modified. Show that the claimed expression for v_{boot} given in the display for this problem is incorrect and find the correct expression. Here $v_{boot} = Var_{\mathbb{F}_n}(T_n)$ where $T_n = \overline{X}_n^2$. [Hint: see Dodd and Korn, *The American Statistician* **61** (2007), 127 - 131, and especially their appendix B, pages 130-131. Apparently the formula given by Wasserman in his problem is from Shao and Tu (1995), page 10; as noted by Dodd and Korn, the expression in Shao and Tu is incorrect.]
 - (b) Explain how the resulting formulas relate to how you would estimate the variance of \overline{X}_n^2 via the delta method.
3. Wasserman, problem 3.8.11, page 41.
4. On page 12, line 4 of Chapter 8 of the lecture notes, it is claimed that if $E_F|X|^r < \infty$ for some $r > 0$, then for the median function $T(F) = F^{-1}(1/2)$ we have

$$nVar_F(T(\mathbb{F}_n)) \rightarrow \frac{1/4}{f^2(F^{-1}(1/2))}.$$

Prove (or disprove) this claim.