Statistics 583, Problem Set 9

Wellner; 5/24/2006

Reading: Wasserman, Chapters 4-9, pages 43-223
Due: Wednesday, May 31, 2006

1. (a) Wasserman, problem 4.7.2, page 59.
   (b) Wasserman, problem 4.7.3, page 59.

2. (a) Consider the kernel density estimator defined in (6.26), Wasserman, page 132. Show that if the density f and the kernel k satisfy the hypotheses of Wasserman's theorem 6.28, page 133, and $h = h_n$ satisfies the hypotheses of Theorem 6.27, then for fixed $x \in \mathbb{R}$,

$$\sqrt{nh_n}(\hat{f}_n(x) - E\hat{f}_n(x)) \to_d N\left(0, f(x)\int k^2(x)dx\right).$$

   (b) Under what restriction on h_n does it follow (from (a) together with further analysis of the bias) that

$$\sqrt{nh_n}(\hat{f}_n(x) - f(x)) \to_d N\left(0, f(x)\int k^2(x)dx\right)?$$

   (c) If $h_n = cn^{-1/5}$ and the hypotheses of (a) hold, find the limiting distribution of $\sqrt{nh_n}(\hat{f}_n(x) - f(x))$.
   (d) Under the same assumptions as in (c), find the limiting distribution of $\sqrt{nh_n}(\sqrt{\hat{f}_n(x)} - \sqrt{f(x)})$.
   (e) Suppose that $x, y \in \mathbb{R}$ with $x < y$. Find the joint limiting distribution of $(\sqrt{nh_n}(\hat{f}_n(x) - f(x)), \sqrt{nh_n}(\hat{f}_n(y) - f(y))$ under the assumptions in (b) and (c).

3. (a) Wasserman, problem 6.9.3, page 143.
   (b) Show that (6.35) on page 136 holds.