Statistics 583, Problem Set 5

Wellner; 4/26/2000

Reminder:	Midterm Exam – next Monday, May 1.
Reading:	Lecture Notes, Chapter 6, pages 25-42;
	Lehmann, TSH (Second Ed.), Chapter 6, pages 282 - 364;
	Ferguson, MS, Chapter 5, Sections 5.6-5.7, pages 242-257.
Due:	Wednesday, May 3, 2000.

1. Let X, Y be independent exponential random variables with parameters μ and ν respectively:
$$p_{\mu,\nu}(x, y) = \mu\nu \exp(-\mu x - \nu y) 1_{(0,\infty)}(x) 1_{(0,\infty)}(y) .$$
Let $\theta = \mu/\nu$. Show that the problem of testing $H : \theta \le 1$ versus $K : \theta > 1$ is invariant under the group of transformations $g_c(x, y) = (cx, cy)$, $c > 0$, and find a UMP invariant test of size α.

2. What happens in the context of problem 1 if we observe $X_1, \ldots, X_m$ i.i.d. as X and $X_1, \ldots, Y_n$ i.i.d. as Y? Can you find a UMP invariant test of H versus K?

3. Suppose that an urn contains N balls with the numbers $z_i = -\log(1 - i/(N + 1))$, $i = 1, \ldots, N$ and we sample $n < N$ balls from this urn. Let $\overline{Y}_n = n^{-1} \sum_1^n Y_i$ denote the sample mean of the sampled valls.
A. Calculate the mean $\mu_N = E(\overline{Y}_n)$ and variance $\sigma_N^2 = Var(\overline{Y}_n)$ of $\overline{Y}_n$.
Find the limits of $\overline{z}_N$ and σ_z^2 as $N \to \infty$.
B. Use the Wald-Wolfowitz-Noether-Hajek finite-sampling CLT to prove that $(\overline{Y}_n - \mu_N)/\sigma_N \to_d N(0, 1)$.
C. What classical two-sample rank statistic is $\overline{Y}_n$ equivalent to under the null hypothesis (of all $X_1, \ldots, X_m, Y_1, \ldots, Y_n$ equal in distribution with a common continuous distribution function F)?