Statistics 583, Problem Set 4

Wellner; 4/19/2000

Reading: Lecture Notes, Chapter 6, pages 25-42;
Lehmann, TSH (Second Ed.), Chapter 6, pages 282 - 364;
Ferguson, MS, Chapter 5, Sections 5.6-5.7, pages 242-257.

Due: Wednesday, April 26, 2000.

1. Suppose that $X_1, \ldots, X_m$ are i.i.d. $F \in \mathcal{F}_c$ and $Y_1, \ldots, Y_n$ are i.i.d. $G \in \mathcal{F}_c$.
A. Find a most powerful similar test of $H_c : F = G \in \mathcal{F}_c$ versus

$$K_1 : F = \text{Exponential}(\mu), \quad G = \text{Exponential}(\nu) \quad \text{with} \quad \nu < \mu \quad \text{known.}$$

B. Does the resulting test have any optimality properties against any composite alternative?
C. If $m = n = 4$ and we observe $\underline{X} = (X_1, X_2, X_3, X_4) = (2.61, 3.02, 1.97, 2.79)$ and $\underline{Y} = (Y_1, Y_2, Y_3, Y_4) = (3.28, 2.19, 2.88, 3.41)$, carry out the test in A at level $\alpha = .1$. What is the approximate $p-$value for the observed data?

2. Consider the testing problem in Problem 1. A. In the context of Problem 1, consider the smaller parametric null hypothesis

$$H_0 : \ F = \text{Exponential}(\mu), \, G = \text{Exponential}(\mu) \, .$$

Find the UMP unbiased test of H_0 versus $K_0 : \nu < \mu$. (This is closely related to problem 2 of problem set #3.)
B. Can you rewrite the UMPU test from part A in such a way that its critical region is asymptotically equivalent to the critical region of the permutation test you found in problem 1? [Hint: you will need to use the WWNH finite sampling CLT.]

3. Consider the critical point $c_\alpha(\underline{Z})$ of the two-sample permutation $t-$test. In class on 4/17 and 4/21 we showed that $c_\alpha(\underline{Z}) \rightarrow_p z_\alpha$ under both the null hypothesis $F = G$ and under the alternative hypothesis $F \neq G$ if $E_F X^2 < \infty$ and $E_G Y^2 < \infty$. What does this imply about the power of the permutation test as $m \wedge n \to \infty$ if $E_F X < E_G Y$?

4. Suppose that $X_1, \ldots, X_n$ are i.i.d. $N(\mu, \sigma^2)$ and consider testing $H : |\mu| \geq 1$ versus $K : |\mu| < 1$. This is the "bioequivalence" problem considered by Perlman and Wu (2000). See (1) on page 356 of Perlman and Wu, and Section 7, pages 361 - 362. Find the likelihood ratio test of H versus K. How is your rejection region related to the set R in (22) of Perlman and Wu?

5. Confidence intervals can be obtained by "inverting" tests, If a test has an optimality property such as being the UMP unbiased test of $H : \theta = \theta_0$ versus $K : \theta \neq \theta_0$, then the corresponding confidence sets often have a corresponding optimality property. Furthermore, note that a family of confidence sets with confidence level decreasing to

zero will define an estimator. See Lehmann, TSH, sections 3.5 and 5.7. The following problem is in this vein. Suppose that $X \sim \text{Binomial}(m, p_1)$ and $Y \sim \text{Binomial}(n, p_2)$. Find the most accurate unbiased confidence interval for the the log-odds ratio

$$\theta \equiv \log\left(\frac{p_2/q_2}{p_1/q_1}\right) .$$

[Hint: See Lehmann, TSH, section 5.7, page 221, for a related example involving comparison of two Poisson parameters.]