

Statistics 583, Problem Set 2

Wellner; 4/5/2000

Reading: Lecture Notes, Chapter 6, pages 11-24;
Lehmann, TSH (Second Ed.), Chapter 4, pages 135 - 185;
Ferguson, MS, Chapter 5, Sections 5.3-5.5, pages 215-242.

Due: Wednesday, April 12, 2000.

1. Suppose that $\mathcal{P} \equiv \{p(x, \theta) : x \in R, \theta \in \Theta \subset R\}$ is a family of density functions with respect to Lebesgue measure such that the mixed second derivative

$$\frac{\partial^2}{\partial \theta \partial x} \log p(x, \theta)$$

exists for all x and θ . Show that if this second partial derivative is ≥ 0 for all x and θ , then \mathcal{P} has monotone likelihood ratio in x .

2. Suppose that $p(x, \theta) = g(x - \theta)$, $x \in R$, $\theta \in R$, where g is a fixed density function on R . Show that this family has monotone likelihood ratio in x if and only if $-\log g$ is convex. [Hint: see Lehmann, TSH, section 9.2, page 509.]

3. (i) Suppose that $X \sim P$ on a measurable space $(\mathcal{X}, \mathcal{A})$. Consider testing the simple hypothesis $H_0 : P = P_0$ versus the simple alternative $H_1 : P = P_1$ based on one observation X from either P_0 or P_1 . Let ϕ be a test of H_0 versus H_1 , and let $a \equiv R(0, \phi) \equiv E_0 \phi(X)$, $b \equiv R(1, \phi) \equiv E_1(1 - \phi(X))$ be the risks associated with a 0 - 1 loss function. Find a test ϕ which minimizes the weighted sum of the risks $c_0 R(0, \phi) + c_1 R(1, \phi) = c_0 a + c_1 b$.

(ii) Show that when $c_0 = c_1$ (both positive), the minimum weighted risk can be expressed in terms of $\int p_0 \wedge p_1 d\mu$ (where p_0, p_1 are densities of P_0, P_1 respectively with respect to a common dominating measure μ), and hence to the total variation distance $d_{TV}(P_0, P_1)$ between P_0 and P_1 .

4. A random variable X takes on the values 1, 2, 3, 4 with probability distribution P_0 or P_1 with mass functions p_0 and p_1 given in the following table:

x	1	2	3	4
$p_0(x)$.1	.2	.3	.4
$p_1(x)$.4	.2	.2	.2

(a) Find a most powerful test of size $\alpha = .2$ for testing P_0 versus P_1 and determine its power.

(b) Find a test ϕ which minimizes the sum of risks $a + b$ where a, b are as defined in the previous problem.

Compute the minimum risk and the total variation distance between P_0 and P_1 .