Statistics 583: Midterm Exam

Wellner; 5/1/2000

Instructions: This exam is to be done with closed notes and closed books.

1. (20 points). **Define** the following terms, providing an appropriate context for your definition:
   A. A permutation test.
   B. Monotone likelihood ratio.
   C. An unbiased test.
   D. A locally most powerful test (for a one-parameter problem).

2. (24 points). **State** any two of the following four theorems, providing an appropriate context in each case:
   A. The Neyman-Pearson lemma.
   B. The generalized Neyman-Pearson lemma.
   C. The Wald - Wolfowitz - Noether - Hájek finite sampling central limit theorem .
   D. The Karlin-Rubin theorem.

3. (36 points). Suppose that $Y_1, \ldots, Y_N$ are independent with $Y_i \sim \text{Binomial}(n_i, p_i)$, $i = 1, \ldots, N$, where
   $$p_i = \frac{1}{1 + \exp(-(\gamma + \delta x_i))}$$
   for given numbers $x_1, \ldots, x_N$. This is a model frequently used in bioassay, where x_i denotes the dose (or logarithm of the does) of a drug given to n_i experimental subjects, and Y_i is the number among these n_i subjects who respond to the drug at the level x_i.
   A. Show that the joint distribution of the Y's is an exponential family. Identify the natural parameters and the sufficient statistics for these parameters. [Hint: first find a simple expression for $p_i/(1 - p_i)$ in terms of γ, δ, and x_i.]
   B. Consider testing $H : \delta \le 0$ versus $K : \delta > 0$. Find the form of the UMP unbiased size α test of H versus K.
   C. What is the distribution of $T \equiv \sum_1^N Y_i$ under $\delta = 0$?
   What is the conditional distribution of $(Y_1, \ldots, Y_N)$ given T under $\delta = 0$?

4. (36 points). Suppose that under the null hypothesis H_c, $X_1, \ldots, X_n$ are i.i.d. $F \in \mathcal{F}_c$, while under the alternative hypothesis K_1, the random variables $X_1, \ldots, X_n$ have joint density h given by

$$h(\underline{x}) = \prod_{i=1}^{n} \lambda_i \exp(-\lambda_i x_i) 1_{(0,\infty)}(x_i)$$

where $\lambda_i = \mu e^{\nu c_i}$ for some fixed numbers (covariates) $c_1, \ldots, c_n$, and constants $\mu > 0$ and $\nu \in R$.

A. Find a most powerful similar test of size α for testing H_c versus K_1. Does your test depend on the values of the constants μ and/or ν?

B. If $n = 3$, $\lambda_1 = 1$, $\lambda_2 = 2$, and $\lambda_3 = 3$, and we observe $(X_1, X_2, X_3) = (3.5, 2.1, .9)$, carry out the test in A at level $\alpha = 1/6$.

C. Find a test which is a locally most powerful similar test of H_c versus $K : \nu > 0$. How does this test differ from the test you found in A?