

Statistics 582, Problem Set 7

Wellner; 2/21/2007

Reading: Chapter 6, sections 6.1-6.3;

Due: Wednesday, February 28, 2006.

1. A random variable X takes on the values 1, 2, 3, 4 with probability distribution $p_0(x)$ or $p_1(x)$ as follows:

x	1	2	3	4
$p_0(x)$.18	.06	.36	.40
$p_1(x)$.36	.18	.24	.22

- A. Find a most powerful test of size $\alpha = .2$ for testing p_0 versus p_1 and determine its power.
 - B. Find a test ϕ which minimizes the sum of risks $a + b$ where $a = E_0\phi$ and $b = E_1(1 - \phi)$.
2. (Problem 3.6, Lehmann and Romano, TSH, page 93.) Suppose that P_0 , P_1 , and P_2 be the probability distributions assigning to the integers 1, ..., 6 the following probabilities:

x	1	2	3	4	5	6
$p_0(x)$.03	.02	.02	.01	0	.92
$p_1(x)$.06	.05	.08	.02	.01	.78
$p_2(x)$.09	.05	.12	0	.02	.72

Determine whether there exists a level- α test of $H : P = P_0$ which is UMP against the alternatives P_1 and P_2 when:

- (i) $\alpha = .01$; (ii) $\alpha = .05$; (iii) $\alpha = .07$.

3. (Problem 3.7, Lehmann and Romano, TSH, page 94.) Suppose that the distribution of X is given by

x	0	1	2	3
$p_\theta(x)$	θ	2θ	$.9 - 2\theta$	$.1 - \theta$

where $0 < \theta < .1$. For testing $H : \theta = .05$ against $\theta > .05$ at level $\alpha = .05$, determine which of the following tests (if any) is UMP:

- (i) $\phi(0) = 1, \phi(1) = \phi(2) = \phi(3) = 0$;
- (ii) $\phi(1) = .5, \phi(0) = \phi(2) = \phi(3) = 0$;
- (ii) $\phi(3) = 1, \phi(0) = \phi(1) = \phi(2) = 0$.

4. Suppose that X_1, \dots, X_n are i.i.d. $N(\theta, \sigma^2)$.

(a) Suppose that $\sigma = \sigma_0$ is known. Consider testing $H : \theta = \theta_0 = 0$ versus $K : \theta = \theta_1 = 1$. In the spirit of chapter 5, plot $(R(\theta_0, \phi), R(\theta_1, \phi))$ for your favorite family of tests ϕ . Find the entire risk body and plot it.

(b) What happens to the risk body as n grows or as $\sigma_0 \rightarrow 0$?

(c) What happens to the risk body as θ_1 decreases toward $\theta_0 = 0$?

(d) What happens to the risk bodies $\{(R(\theta_0, \phi), R(\theta_{1,n}, \phi)) : n \geq 1\}$ when $\theta_1 \equiv \theta_{1,n} \equiv \theta_0 + cn^{-1/2}$?

5. **Optional bonus problem:** Suppose that $X_i \sim N(i\Delta, 1)$, $i = 1, \dots, n$ are independent. Show that there exists a UMP test of $H : \Delta \leq 0$ versus $K : \Delta > 0$, and determine it as explicitly as possible.