Risk Plots: MLE versus Minimax

Jon A. Wellner

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As we showed in Example 6.1, for estimating θ given observation of $X \sim$ Binomial(n, θ), the minimax rule is given by

$$d_M(X) = \frac{1}{1+\sqrt{n}} \frac{1}{2} + \frac{\sqrt{n}}{1+\sqrt{n}} \frac{X}{n},$$

while the MLE of θ is $\hat{\theta} \equiv X/n$. Thus the risk function of d_M satisfies

$$R(\theta, d_M) = \frac{1}{4(1+\sqrt{n})^2} \le \frac{\theta(1-\theta)}{n} = R(\theta, \hat{\theta}_n)$$

for

$$|\theta - 1/2| \le \frac{1}{2} \frac{\sqrt{1+2\sqrt{n}}}{1+\sqrt{n}} \equiv c_n \sim \frac{1}{\sqrt{2}n^{1/4}};$$

or equivalently for $\theta \in [1/2 - c_n, 1/2 + c_n] \equiv [A_n, B_n]$. Table 1 gives these intervals for $n = 2, 3, 5, 10, 100, 500$:

n	A_n	B_n
2	.095	.905
3	.113	.887
5	.139	.861
10	.175	.825
100	.292	.708
500	.355	.645

Here is a plot comparing $nR(\theta, \hat{\theta}_n) = \theta(1-\theta)$ to $nR(\theta, d_M) = n/(4(1+\sqrt{n})^2)$ for sample sizes $n = 2, 3, 5, 10, 100, 500$.

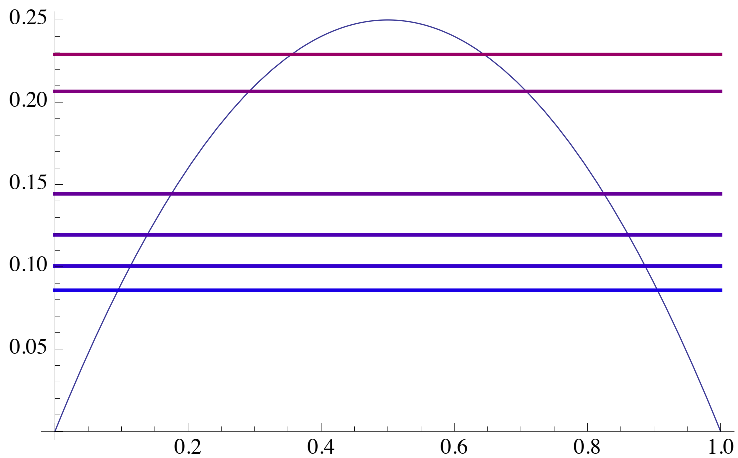

Figure 1: Risks compared, binomial (n, θ): minimax versus MLE for $n = 2,\ 3,\ 5,\ 10,\ 100, 500$