Statistics 581, Problem Set 10

Wellner; 12/3/2010

Reading: Chapter 4, Sections 1-4;
Ferguson, ACLST, Chapter 20, pages 133-139; Chapter 22, pages 144-150;
Lehmann and Casella, Chapter 6, especially section 6.5, pages 461-468.
Due: Friday, December 10, 2010.
Reminder: Final Exam; Monday, December 13, 2010.

1. (a) Ferguson, ACLST, page 139, problem 3.
   (b) What if Ferguson's density $f(x|\theta)$ with $\theta \in (0,1)$ is replaced by $\theta = (\gamma, \eta) \in (0,1) \times (0,\infty)$ and
   $$f(x|\theta) \equiv f(x|\gamma,\eta) = \{(1-\gamma)e^{-x} + \gamma\eta^2 x \exp(-\eta x)\} 1_{[0,\infty)}(x)?$$
   Can you estimate γ and η by the method of moments? Can you improve method of moment estimators via one-step estimators?

2. Ferguson, ACLST, page 118, problem 3. (See also Example 4.3.7, page 21, Chapter 4 notes.)

3. Lehmann and Casella, problem 6.8, page 509.

4. Lehmann and Casella, problem 6.9, page 509.

5. Ferguson, ACLST, page 149, problem 2 modified as follows:
   (a) Find the LR test statistic of the null hypothesis $H_0 : \mu = c\theta$ for any fixed number $c > 0$, and find the asymptotic distribution of the LR statistic under H_0.
   (b) Does the theory of our chapter 4 (or Ferguson's chapter 22) apply directly?
   (c) Does the local asymptotic power of your test depend on c?

6. **Optional bonus problem 1.**
   Ferguson, ACLST, page 150, problem 3. Does the theory in our chapter 4 (or Ferguson's chapter 22) apply directly?

7. **Optional bonus problem 2:** Lehmann and Casella, problem 6.10, page 510.

8. **Optional bonus problem 3:** Suppose that (as in Lemma 5.2, page 38, Chapter 3 Notes) P and Q are two probability measures on a measurable space $(\mathcal{X}, \mathcal{A})$ with densities p and q with respect to a $\sigma-$finite dominating measure μ, and P^n and Q^n denote the corresponding product measures on $(\mathcal{X}^n, \mathcal{A}_n)$ (of $X_1, \ldots, X_n$ i.i.d. as P or Q respectively).
(a) What is the relationship between $K(P^n, Q^n)$ and $K(P, Q)$, if any?
(b) If P is the Normal$(0, \sigma^2)$ distribution and Q is the Normal(μ, σ^2) distribution, compute $K(P, Q)$, $\rho(P, Q) = \int \sqrt{pq} d\mu$, and $H^2(P, Q)$.
(c) Use the results of (a) and (b) together with Lemma 5.2 to calculate $K(P^n, Q^n)$, $\rho(P^n, Q^n)$, and $H^2(P^n, Q^n)$ when P and Q are as in (b).
(d) Find a sequence μ_n so that, with Q_n being the Normal distribution with mean μ_n, the quantities $K(P^n, Q_n^n)$, $\rho(P^n, Q_n^n)$, and $H^2(P^n, Q_n^n)$ converge to finite limits as $n \to \infty$.