Statistics 581, Problem Set 9

Wellner; 11/25/2009

Reading: Chapter 4, Sections 1-2;
Ferguson, ACLST, Chapters 18- 20, pages 119-125, 133-139; Chapter 22, pages 144-150;
Lehmann and Casella, Chapter 6, especially section 6.5, pages 461-468.
Due: Wednesday, December 2, 2009.

1. Lehmann and Casella, TPE, problem 6.3.22, page 503, reworded as follows. (In other words, prove (vi) of theorem 1.2, pages 5-6, chapter 4 notes). Suppose that $X_1, \ldots, X_n$ are i.i.d. with density p_θ, $\theta \in \Theta \subset R^k$, satisfying the hypotheses of theorem 4.1, page 463 (the Cramér conditions given in (A) - (D) on pages 462-463). Show that the following Local Asymptotic Normality (LAN) result holds for the (local) log- likelihood ratios: with

$$L_n(\theta) \equiv \log(\prod_{i=1}^n p_\theta(X_i)) = \sum_{i=1}^n \log p_\theta(X_i),$$

for a fixed $\theta_0 \in \Theta$,

$$\begin{aligned} L_n(\theta_0 + n^{-1/2}\underline{t}) - L_n(\theta_0) &= \frac{1}{\sqrt{n}} \sum_{i=1}^n \underline{t}^T \dot{\underline{l}}_\theta(X_i) - \frac{1}{2}\underline{t}^T I(\theta_0)\underline{t} + o_p(1) \\ &\to_d N(0, t^T I(\theta_0)t) - \frac{1}{2} t^T I(\theta_0) t \\ &\overset{d}{=} N(-\sigma^2/2, \sigma^2) \end{aligned}$$

under P_{θ_0} where $\sigma^2 \equiv \underline{t}^T I(\theta_0)\underline{t}$. (The convergence in the last display actually holds under the considerably weaker hypothesis of Hellinger differentiability of p_θ at θ_0, as stated in Corollary 3 of section 3.3, page 28, of the Chapter 3 notes.)

2. Lehmann and Casella, problem 6.2.14, page 501. [**Hint:** We did this in class on Friday 11/20 using the first display on page 26 of the Chapter 3 notes. What remains to prove is uniform integrability or some other justification of the interchange of limit and integration.]

3. (a) Exercise 2.1.6, page 10, chapter 2 notes.
   (b) Exercise 2.1.7, page 10, chapter 2 notes.

4. **Optional bonus problem:** Lehmann and Casella, problem 6.3.18, page 502. [**Note:** It seems to me that 3.15(b) should be 3.15(c) since $C(0, a)$ is a *scale family*).]