Statistics 581, Problem Set 1

Wellner; 9/24/2008

Reading: Lehmann & Casella, TPE, pages 1 - 32; Ferguson, ACILST, Chapter 1, pages 3-7; skim Chapter 0 handout; read Chapter 1 handout.
Due: Wednesday, October 1, 2008.

1. Let X and Y be i.i.d. Uniform$(0,1)$ random variables Define $U = X - Y$, $V = \min(X,Y) = X \wedge Y$.

   (i) What is the range of (U,V)?

   (ii) Find the joint density function $f_{U,V}(u,v)$ of the pair (U,V). Are U and V independent?

2. Lehmann & Casella, TPE, problem 5.33, page 69.
   (c) Find the papers by Morris (1982, 1983b). In what sense do the the normal binomial, Poisson, gamma, and negative binomial families have "quadratic variance functions"?

3. Lehmann and Casella, TPE, problem 3.8, page 64 (with $W = 1_{[X \le Y]}$). Now do the problem for X with distribution function F independent of Y with distribution function G. Does the independence of Z and W hold in general?

4. Suppose that $X \sim$ Uniform$(0,1)$ and $Y = (1 - X^2)^{-1}$. Find the joint distribution function $F(x,y) = F_{X,Y}(x,y)$ of (X,Y). Does the pair (X,Y) have a joint density function (with respect to Lebesgue measure)?

5. Ferguson, ACILST, #1, page 6.

6. (a) Lehmann and Casella, TPE, problem 1.4, page 62.
   (b) Lehmann and Casella, TPE, problem 1.10, page 62.