Statistics 581, Problem Set 1

Wellner; 9/27/2006

Reading: Lehmann & Casella, TPE, pages 1 - 32; skim Chapter 0 handout; read Chapter 1 handout.
Due: Wednesday, October 4, 2005.

1. Let X and Y be i.i.d. Uniform$(0,1)$ random variables Define $U = X - Y$, $V = \max(X,Y) = X \vee Y$.

   (i) What is the range of (U,V)?

   (ii) Find the joint density function $f_{U,V}(u,v)$ of the pair (U,V). Are U and V independent?

2. Lehmann & Casella, TPE, problem 5.33, page 69.

3. (a) Lehmann & Casella, TPE, problem 3.5, page 64.
   (b) Lehmann & Casella, TPE, problem 3.6, page 64.
   (c) Lehmann & Casella, TPE, problem 3.7, page 64.

4. Suppose that $X \sim$ Uniform$(0,1)$ and $Y = -\log(1-X)$. Find the joint distribution function $F(x,y) = F_{X,Y}(x,y)$ of (X,Y).

5. Ferguson, ACILST, #6, page 7. (This is known as the Polya-Cantelli lemma; see Chapter 2, Propostion 2.11, page 10.)

6. (a) Lehmann and Casella, TPE, problem 1.2, page 62.
   (b) Lehmann and Casella, TPE, problem 1.3, page 62.