Statistics 581, Problem Set 10

Wellner; 11/29/2000

Reading: Chapter 4, Sections 1-4;
Ferguson, ACLST, Chapter 20, pages 133-139; Chapter 22, pages 144-150;
Lehmann and Casella, Chapter 6, especially section 6.5, pages 461-468.
Due: Wednesday, December 6, 2000.

1. Ferguson, ACLST, page 149, problem 2 modified as follows:
(a) Find the LR test statistic of the null hypothesis $H_0 : \mu = c\theta$ for any fixed number $c > 0$, and find the asymptotic distribution of the LR statistic under H_0.
(b) Does the theory of our chapter 4 (or Ferguson's chapter 22 apply directly?
(c) Does the local asymptotic power of your test depend on c?

2. Ferguson, ACLST, page 150, problem 3. Does the theory in our chapter 4 (or Ferguson's chapter 22) apply directly?

3. Suppose that $(Y|Z) \sim \text{Poisson}(\lambda e^{\gamma Z})$, and $Z \sim Bernoulli(\eta)$, and $\theta = (\lambda, \gamma, \eta)$. Let $X = (Y, Z)$, and suppose that we observe $X_1, \ldots, X_n$ i.i.d. as X.
Consider testing the hypothesis $H : \gamma = 0$ versus $K : \gamma \neq 0$. (Note that the null hypothesis is *not simple*, but *composite*; the values of λ and η are not specified by the hypothesis H.)
(a) Propose three different test statistics for testing H versus K, and briefly discuss how you would compute them.
(b) Do our results in Chapter 4 apply to the (asymptotic) distribution under H of the test statistics you proposed in (a)?
(c) Consider local alternatives of the form $\gamma_n = tn^{-1/2}$ for $t \in R$ fixed. Give an expression for the local asymptotic power of the tests you proposed in (a) for these alternatives.
(d) Suppose that $\gamma_1 \neq 0$ is the "true" value of the parameter γ. Show that the test statistics you proposed in (a), when appropriately normalized, converge in probability to positive constants, and identify these constants as explicitly as possible.