**Statistics 581, Problem Set 1**

Wellner; 9/27/00

**Reading:** Lehmann & Casella, TPE, pages 1 - 32; all of Chapter 0 handout; start reading Chapter 1 handout.
**Due:** Wednesday, October 4, 2000.

1. Let $X$ and $Y$ be i.i.d. Uniform$(0,1)$ random variables Define $U = X+Y$, $V = \min(X,Y) = X \wedge Y$.

   **(i)** What is the range of $(U,V)$?

   **(ii)** Find the joint density function $f_{U,V}(u,v)$ of the pair $(U,V)$. Are $U$ and $V$ independent?

2. Prove part (ii) of Proposition 1.1: There exists a minimal field, $\sigma-$field, and monotone class generated by any class of subsets of $\Omega$.

3. (a) Suppose that $\{\mathcal{A}_n\}$ is an increasing sequence of fields, i.e. $\mathcal{A}_n \subset \mathcal{A}_{n+1}$ for all $n \ge 1$. Show that $\cup_{n=1}^{\infty}\mathcal{A}_n$ is a field. (b) Suppose that the $\mathcal{A}_n$ of (a) are $\sigma-$ fields. Show by constructing a counterexample that $\cup_{n=1}^{\infty}\mathcal{A}_n$ need not be a $\sigma-$field.

4. Let $\mu$ be a finite measure on $R$, and let $G(x) = \mu((-\infty, x])$. Show that

$$\int [G(x+c) - G(x)]dx = c\mu(R) .$$

   [Hint: use Fubini's theorem.]

5. Let $\mathcal{X} = (0,1)$, $\mathcal{Y} = (1,\infty)$, both equipped with the Borel sets and Lebesgue measure. Let $f(x,y) = e^{-xy} - 2e^{-2xy}$. Show that:
   (a) $\int_0^1 (\int_1^{\infty} f(x,y)dy)dx = \int_0^1 x^{-1}(e^{-x} - e^{-2x})dx$ exists and is $> 0$.
   (b) $\int_1^{\infty} (\int_0^1 f(x,y)dx)dy = \int_1^{\infty} x^{-1}(e^{-2y} - e^{-y})dy$ exists and is $< 0$.
   (c) Why does Fubini's theorem fail here?

6. Lehmann, TPE, problem 1.3, page 62.