STATISTICS 581:
Advanced Theory of Statistical Inference
Fall, 2017

Tentative Outline

1. **Chapter 0: Measures, Integration, Convergence.**
    - Section 0.1: Measures.
    - Section 0.2: Measurable functions and integration.
    - Section 0.3: Absolute continuity, Radon-Nikodym theorem, Fubini's theorem.
2. **Chapter 1: Special Distributions.**
    - Special distributions.
    - Convolution and related formulas.
    - The multivariate normal distribution.
    - Integration by parts formulas.
3. **Chapter 2: Some basic large sample theory.**
    - Modes of convergence.
    - Classical limit theorems.
    - Replacing $\rightarrow_d$ by $\rightarrow_{a.s.}$: Skorokhod's theorem.
    - Empirical distributions, empirical measures, empirical processes, and Brownian bridge.
    - The partial sum process and Brownian motion.
    - Sample quantiles and asymptotic linearity of estimators.

4. **Chapter 3: Lower bounds for estimation.**

   - Introduction and examples.
   - Cramér - Rao bounds for parametric models.
   - Regular estimates; superefficiency; LAN and Le Cam's three lemmas.
   - Hajek's convolution theorem and local asymptotic minimax (LAM) theorem.

5. **Chapter 4: Efficient Likelihood Estimation and Related Tests.**

   - Maximum likelihood and efficient likelihood estimation.
   - Likelihood ratio, Wald, and Rao (or score) tests.
   - Examples.
   - Consistency of maximum likelihood estimates.
   - The EM algorithm and related methods.
   - Nonparametric MLE.
   - Limit theory for the statistical agnostic: $P \notin \mathcal{P}$.