

STATISTICS 581:
Advanced Theory of Statistical Inference
Fall, 2006

Tentative Outline

1. **Chapter 0: Measures, Integration, Convergence.**
 - Section 0.1: Measures.
 - Section 0.2: Measurable functions and integration.
 - Section 0.3: Absolute continuity, Radon-Nikodym theorem, Fubini's theorem.

2. **Chapter 1: Special Distributions.**
 - Special distributions.
 - Convolution and related formulas.
 - The multivariate normal distribution.
 - Integration by parts formulas.

3. **Chapter 2: Some basic large sample theory.**
 - Modes of convergence.
 - Classical limit theorems.
 - Replacing \rightarrow_d by $\rightarrow_{a.s.}$: Skorokhod's theorem.
 - Empirical distributions, empirical measures, empirical processes, and Brownian bridge.
 - The partial sum process and Brownian motion.
 - Sample quantiles and asymptotic linearity of estimators.

4. **Chapter 3: Lower bounds for estimation.**

- Introduction and examples.
- Cramér - Rao bounds for parametric models.
- Regular estimates; superefficiency; LAN and Le Cam's three lemmas.
- Hajek's convolution theorem and local asymptotic minimax (LAM) theorem.

5. **Chapter 4: Efficient Likelihood Estimation and Related Tests.**

- Maximum likelihood and efficient likelihood estimation.
- Likelihood ratio, Wald, and Rao (or score) tests.
- Examples.
- Consistency of maximum likelihood estimates.
- The EM algorithm and related methods.
- Nonparametric MLE.
- Limit theory for the statistical agnostic: $P \notin \mathcal{P}$.