Statistics 523, Problem Set 6

Wellner; 5/5/99

Reading: Shorack, PfS; Chapter 12, pages 247 - 283.

Due: Wednesday, May 12, 1999.

1. Let $\tau_a \equiv \inf\{t > 0 : \mathbb{S}(t) = a > 0\}$ where $\mathbb{S}$ is standard Brownian motion. Define $\phi(s, a) \equiv E\exp(-s\tau_a)$. Use the strong Markov property of $\mathbb{S}$ to show that for $a, b > 0$ we have $\phi(s, a + b) = \phi(s, a)\phi(s, b)$. Use this to deduce via scaling properties of $\mathbb{S}$ that $\phi(s, a) = \exp(-c\sqrt{s}a)$ for some $c > 0$. [Note problem 37.10, Billingsley, page 550; or Remark 8.3, page 96, Karatzas and Shreeve (1991).]

2. (a) What is the relationship of $\sup_{0\le s\le t} \mathbb{S}(s)$ to τ_a in problem 1.
   (b) Use this to find the exact distribution of τ_a, and then use this to check the computation of $\phi(s, a) = E\exp(-s\tau_a)$ of problem 1.
   (c) Show that $E(\tau_a) = \infty$
   (d) Show that $\tau_a =_d a^2/Z^2$ where $Z \sim N(0, 1)$.

3. PfS, Exercise 12.9.1, page 273.