

Statistics 523, Problem Set 4

Wellner; 4/21/99

Reading: Shorack, PfS; Chapter 15, pages 335 - 348.
Chapter 12, pages 247 - 283.

Due: Wednesday, April 28, 1999.

1. PfS, Exercise 15.1.5, page 340.
2. PfS, Exercise 15.1.7, page 340.
3. PfS, Exercise 15.4.2, page 347.
4. Due on May 5: an outline of your project paper.
5. **Bonus Problem:** (Renewal process CLT). Let X_1, X_2, \dots be i.i.d. positive random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$. Let $S_n = X_1 + \dots + X_n$, and $N_t = \sup\{m : S_m \leq t\} = \sum_{j=1}^{\infty} 1_{[0,t]}(S_j)$. (The process $\{N_t : t \geq 0\}$ is called a *renewal process*. Show that

$$\sqrt{n} \left(\frac{N_n}{n} - \frac{1}{\mu} \right) \rightarrow_d \frac{\sigma}{\mu^{3/2}} Z \sim N\left(0, \frac{\sigma^2}{\mu^3}\right).$$

Hint: use a random sample size CLT.