Statistics 523, Problem Set 5 Bonus

Wellner; 5/6/2020

Due: Wednesday, May 13, 2020.

Reading: Durrett, *Stochastic Calculus*, chapters 2-3.

Reminder: Midterm Exam Due, Friday, 8 May

1. Durrett, Exercise 3.1, page 43: If X is continuous and locally of bounded variation then $t \mapsto V_t$ is continuous. Here V_t is the variation of X on $[0, t]$.
2. Durrett, Exercise 3.6, page 52: Show that if X_t is a bounded martingale, then $X_t^2 - \langle X \rangle_t$ is a uniformly integrable martingale.
3. Durrett, Exercise 3.8, page 52: if $S \le T$ are stopping times and $\langle X \rangle_S = \langle X \rangle_T$, then X is constant on $[S, T]$.
4. Durrett, Exercise 3.9, page 52: Conversely, if $S \le T$ are stopping times and X is constant on $[S, T]$, then $\langle X \rangle_S = \langle X \rangle_T$.
5. Durrett, Exercise 4.2, page 56: $\|H\|_X^2$ is a norm.
6. Durrett, Exercise 4.3, page 56: $X \in \mathcal{M}^2$ if and only if $EX_0^2 < \infty$ and $E\langle X \rangle_\infty < \infty$.
7. Durrett, Lemma 3.7, page 49: fill in the details of the proof of this Lemma.