

Statistics 523, Problem Set 2

Wellner; 4/8/20

Reading: Shorack, PFS (2017), Chapter 9, pages 194-224;
Shorack, PFS (2017), Chapter 10, pages 225-263.

Due: Wednesday, April 15, 2020.

1. Exercise 12.9.3, PFS (2017) page 328: Prove theorem 9.1(b) when all $t_{n,k} = k/2^n$.
[Hint: Use the Paley-Zygmund inequality.]
2. Suppose that $\xi_1, \xi_2, \dots, \xi_n$ are i.i.d. Uniform(0, 1) rv's, let $\xi_{n:0} \equiv 0 \leq \xi_{n:1} \leq \xi_{n:2} \leq \dots \leq \xi_{n:n} \leq 1 \equiv \xi_{n:n+1}$ be the order statistics, and let $\delta_{n:i} \equiv (\xi_{n:i} - \xi_{n:i-1})$, $i \in \{1, \dots, n+1\}$, be the spacings. Give a direct proof of the fact that $\sqrt{n} \max_{1 \leq i \leq n+1} \delta_{n:i} \rightarrow_p 0$. For what sequences $c_n \nearrow \infty$ do we have $c_n \max_{1 \leq i \leq n+1} \delta_{n:i} = O_p(1)$?
3. Suppose that Y_1, Y_2, \dots, Y_{n+1} are i.i.d. Exponential (1) random variables, and let $S_k \equiv \sum_{j=1}^k Y_j$ for $1 \leq k \leq n+1$. In the Skorokhod embedding theorem proof it was claimed that

$$\left(\frac{S_1}{S_{n+1}}, \frac{S_2}{S_{n+1}}, \dots, \frac{S_n}{S_{n+1}} \right) \stackrel{d}{=} (\xi_{n:1}, \dots, \xi_{n:n}).$$

Prove this.

4. Suppose that $g(x) = \int_0^1 f_0(t)x(t)dt$ for some fixed absolutely continuous function f_0 on $[0, 1]$ with $\int_0^1 [f_0'(t)]^2 dt < \infty$. What does Strassen's theorem say about $\limsup_{n \rightarrow \infty} g(\mathbb{Z}_n)$ where $\mathbb{Z}_n \equiv \mathbb{S}(n \cdot) / \sqrt{2n \log \log n}$ and \mathbb{S} is a fixed Brownian motion process on $[0, \infty)$.
5. PFS (2017), Exercise 9.3.6; page 205.
Let ϕ be a chf. Show that $c^{-1} \int_0^c \phi(tu) du$ is a chf.
6. Bonus question 1: PFS (2017), Exercise 9.3.4, page 205).
Derive the Logistic(0, 1) characteristic function. Hint: use lemma 9.3.2.