Statistics 523, Problem Set 1

Wellner; 4/1/2020

Reading:

Shorack, PfS 2017	Section 8.7, pages 179 - 180
Shorack, PfS 2017	Sections 12.5 - 12.8, pages 312 - 326

Due: Wednesday, April 8, 2020

1. Consider the bivariate distribution function H defined in the proof of Skorokhod's theorem in the course notes (and PfS, 2017 page 316). Show that

$$H(a,b) \equiv \int_{[0,a]} \int_{(0,b]} \frac{u+v}{EX^+} dF(-u) dF(v)$$

is in fact a bivariate d.f. on $[0, \infty) \times (0, \infty)$.

2. Consider sampling n balls from an urn with R red balls and W white balls.
(a) Show that if the sampling is carried out with replacement, the process which counts the number of red balls drawn is a Markov process.
(b) Is the process defined in (a) a strong Markov process?
(c) Now suppose that the sampling is without replacement. Is the process which counts the number of red balls in the sample a Markov process? Justify your answer.

3. PfS Exercise 12.1.6, page 299, parts (a) and (b).

4. PfS (2017), Exercise 12.8.1, page 324: let $X_0 \equiv 0$ and let $X_1, X_2, \ldots$ be i.i.d. with mean zero and variance $\sigma^2 \equiv E(X_1^2) < \infty$. Let $S_k \equiv X_1 + \cdots + X_k$ for each integer $k \geq 0$.
(a) Find the asymptotic distribution of $S_1 + \cdots + S_n)/c_n$ for an appropriate sequence c_n.
(b) Determine a representation for the asymptotic distribution of the "absolute area" under the partial sum process as given by $|S_1| + \cdots + |S_n|/c_n$.

5. State the tentative topic or title of your Project / Paper and several key references. A tentative outline will be due on Wednesday, May 4. The Project / Paper itself will be due on Monday, June 8.

6. **Bonus problem 1:** PfS (2017) Exercise 12.8.2, page 325.

7. **Bonus problem 2:** Suppose that the process X in the context of Theorem 12.5.1 is a Markov process. Can you give an example of such a (Markov) process that is not a strong Markov process?