Statistics 523, Problem Set 6

Wellner; 5/7/2010

Reading: Shorack, PfS; Chapter 13, sections 5 and 6.

Due: Friday, May 14, 2010.

1. PfS, Exercise 13.7.1, page 382.
2. PfS, Exercise 13.7.2, page 382.
3. PwM, E10.5, page 233.
4. Let $X_1, X_2, \ldots$ be i.i.d. Rademacher random variables (so that $P(X_i = \pm 1) = 1/2$), and let $S_n = \sum_{j=1}^n$ be the associated partial sum (or simple random walk) process. Let $\mathcal{A}_n$ be the sigma-field generated by $S_1, S_2, \ldots, S_n$. Now $\{S_n, \mathcal{A}_n\}_{n=0}^\infty$ is a martingale, and $\{|S_n|, \mathcal{A}_n\}$ is a sub-martingale. Find a predictable increasing process $\{A_n\}$ so that with $Y_n \equiv |S_n| - A_n$, $\{Y_n, \mathcal{A}_n\}$ is a martingale. You should compute A_n as explicitly as possible in terms of the X_j's.
5. **Optional bonus problem:** What is the analogue of problem 4 above with $\{S_n, \mathcal{A}_n\}$ replaced by $\{B_t, \mathcal{A}_t\}_{t \geq 0}$ where B is Brownian motion (with its natural self-induced filtration)?