Statistics 523, Problem Set 3

Wellner; 4/16/2010

Reading: Shorack, PfS; Chapter 13, pages 349 - 376;
Williams, PwM, Chapters 10, 11, and 12.

Due: Friday, April 23, 2010.

1. Let $R \equiv \inf\{t > 1 : \ B_t = 0\}$ for Brownian motion B (started from $x \in \mathbb{R}$ at time $t = 0$), and let $L = \sup\{t \le 1 : \ B_t = 0\}$.
(i) Use the (ordinary) Markov property of Brownian motion to show that
$$P_x(R > 1 + t) = \int p_1(x, y) P_y(\tau_0 > t) dy, \qquad \text{and}$$
$$P_0(L \le t) = \int p_t(0, y) P_y(\tau_0 > 1 - t) dy$$
where $\tau_0 \equiv \inf\{t > 0 : B_t = 0\}$ and $p_t(x, y) = \phi((y - x)/\sqrt{t})/\sqrt{t}$ is the transition density for Brownian motion.
(ii) Use the distribution of τ_b we computed in class and in problem 4 of problem set 2 to show that
$$P_0(L \le s) = (2/\pi) \arcsin(\sqrt{s}),$$
and find the corresponding result for R.

2. Let B denote standard Brownian motion starting from 0. Consider the stochastic process $\{\tau_b : b > 0\}$. Show that this process has stationary and independent increments. How is it related to the process $M_t \equiv \sup_{0 \le s \le t} B_s$?

3. Exercise 12.7.3(a), PfS, page 325. (This is connected to Example 13.1.8, page 352.)

4. Exercise 13.1.6, PfS, page 353.