Statistics 523, Problem Set 1, Corrected Version

Wellner; 4/2/10

Reading: Shorack, PfS, Chapter 5, pages 98-101; Chapter 8, pages 179-180; Chapter 12, pages 301-311;
Williams, PwM, Chapters 10.8, pages 97-98.

Due: Friday, April 9, 2010.

1. Let $X_1, X_2, \ldots$ be i.i.d. $U(0,1)$, let $S_n = X_1 + \cdots + X_n$ and let $T \equiv \inf\{n : S_n > 1\}$. Show that $P(T > n) = 1/n!$ and hence that $E(T) = e$ and $E(S_T) = e/2$.

2. Let $\mathcal{A}_k$ be an increasing family of $\sigma-$fields, $\mathcal{A}_1 \subset \mathcal{A}_2 \subset \mathcal{A}_3 \subset \cdots$. Then N is a stopping time relative to $\{\mathcal{A}_k\}$ if $[N = k] \in \mathcal{A}_k$ for all $k \geq 1$. Let N_1, N_2 be stopping times relative to $\{\mathcal{A}_k\}$. Show that $N_1 \wedge N_2$, $N_1 \vee N_2$, and $N_1 + N_2$ are stopping times relative to $\{\mathcal{A}_k\}$.

3. In the proof of the existence of Brownian motion as a continuous process on 4/2, we used a 4th moment version of Markov's inequality and reached the conclusion the Brownian motion is Hölder continuous with exponent $\gamma < 1/4$. Can this be strengthened by using a higher-order version of Markov's inequality? What can be concluded if the 4th moment bound is replaced by an 8th moment bound, or by a $2m$th moment bound, or by an exponential bound? (The moment formula given in PfS (12.3.6), page 308 is relevant in this connection.)

4. Exercise 12.3.1, PfS, page 309. In the proof of (17), that $t\mathbb{S}(1/t)$ is a Brownian motion, can you use a weaker result than the LIL of (10) to conclude continuity of the sample paths at 0?