

Statistics 522, Problem Set 1

Wellner; 1/6/99

Reading: Shorack, PFS, Chapter 10, pages 179 - 213;
Kallenberg, FMP, Chapter 1, pages 1-6, and Appendix 1, pages 455-457.

Due: Wednesday, January 13, 1999.

1. Complete the proof of (6) in Inequality 3.4, page 181, PFS, i.e. show how the two one-sided arguments combine to yield the inequality with absolute value signs.
2. Exercise 10.3.1, page 182, PFS.
3. Suppose that X is a random variable, and $X' \stackrel{=}{=} X$ is independent of X , so that $X - X'$ is symmetric. Show that

$$P(|X| \leq a)P(|X'| > t + a) \leq P(|X - X'| > t),$$

and hence, by choosing a so that $P(|X| \leq a) > 1/2$,

$$P(|X| > t + a) \leq 2P(|X - X'| > t) \leq 4P(|X| > t/2).$$

Use this last display to show that $E|X - X'|^r < \infty$ if and only if $E|X|^r < \infty$.

4. (A weak law of large numbers under the assumption of uncorrelated summands.) Suppose that X_1, X_2, \dots are uncorrelated and $E(X_j^2) \leq M < \infty$ for all $j \geq 1$. Show that $\overline{X} - E(\overline{X}_n) = (S_n - ES_n)/n \rightarrow_p 0$ and $\overline{X}_n - E(\overline{X}_n) \rightarrow_2 0$ as $n \rightarrow \infty$.
5. (L_1 -convergence in the SLLN.) Suppose that X_1, \dots, X_n, \dots are i.i.d with $E|X_1| < \infty$. Show that $\overline{X}_n \rightarrow_1 \mu \equiv E(X_1)$; i.e. $E|\overline{X}_n - \mu| \rightarrow 0$ as $n \rightarrow \infty$. [Hint: show that $|\overline{X}_n| \leq Y_n$ where Y_n is uniformly integrable, and that this implies the uniform integrability of \overline{X}_n .