

Statistics 522
Wellner, 1/11/99

Typos and Suggested Minor Revisions
for *Probability for Statisticians*
by Galen R. Shorack

Chapter 1.

1. page 3: Prop 1.1.(j) equation (4): change “subsets of \mathcal{A} ” to “subsets of Ω ”.
2. page 3: Definition 1.3.(j): Maybe “A subset A of Ω ” or “An arbitrary subset A of Ω ” would be better.
3. page 4: last line of “Motivation 1.2”, just before Example 1.1: change “subets” to “subsets”.
4. page 6: proof of proposition 1.3: in (a), change $\sum_1^n A_n$ to $\sum_1^n A_k$.
5. page 9: Proof, part (b), in the step from 2nd line $\mu^*(TB_n) + \mu^*(TA^c)$ to the 3rd line $\mu^*(TB_nA_1) + \dots$, the \geq can be replaced by $=$. \geq is true and doesn't hurt, but $=$ is also true.
6. page 9: Proof, part (b) just below the preceding item, change $A \in \mathcal{A}$ to $A \in \mathcal{A}^*$.
7. page 14, line -4: change “fine” to “finite”.

Chapter 2.

1. page 22, line -3: change $[-\infty, b) = \cup_{n=1}^{\infty} [-\infty, b + 1/n]$ to $[-\infty, b) = \cup_{n=1}^{\infty} [-\infty, b - 1/n]$.
2. page 24, line -6: change “Suppose $(\Omega, \mathcal{A}, \mu)\dots$ ” to “Suppose $X : (\Omega, \mathcal{A}, \mu)\dots$ ”
3. page 25, Proposition 2.5: change “Suppose that Z is a rv” to “Suppose that Z is a measurable function”.

4. page 25, -14: change “hold” to “holds”.
5. page 25, Exercise 2.2 (Verification criterion): change (18) to: if $V_n \in \mathcal{V}$ satisfy $V_n \uparrow V$, then $V \in \mathcal{V}$; in particular, if $A_n \nearrow A$ with $1_{A_n} \in \mathcal{V}$, then $1_A \in \mathcal{V}$.
6. page 27, exercise 2.3.1: in the first line, change “ (Ω, \mathcal{A}, P) ” to “ $(\Omega, \mathcal{A}, \mu)$ ”.
7. page 28, (5): change $X_n \rightarrow_{a.s.}$ to $X_n \rightarrow_{a.e.}$; change $|X_m - X_n|$ to $|X_m - X|$; also make the second change in (6).
8. page 31, line 1: change “sueful” to “useful”.
9. page 31, line 5: change “probvability” to “probability”.

Chapter 3.

1. page 37, line 4: change (7) to read as follows:

$$\int 2Y = \int \liminf(2Y - Z_n) \leq \liminf \int (2Y - Z_n) = \int 2Y - \limsup \int Z_n,$$

and hence $\limsup \int Z_n d\mu \leq 0$.

2. page 38, line 4: either reverse the inequality in the middle of this display, or (better!) change the display to

$$\begin{aligned} \int (2Y - 0) &= \int \liminf(2Y - Z_n) \leq \liminf \int (2Y - Z_n) \\ &= \liminf \left(\int 2Y - \int Z_n \right) = \int 2Y - \limsup \int Z_n, \end{aligned}$$

and then simplify the conclusion in the next line to just $\limsup \int Z_n d\mu \leq 0$.

3. page 39, line -2: change “Let $\mu_0 = \mu|_{\mathcal{A}}$ ” to “Let $\mu_0 = \mu|_{\mathcal{A}_0}$ ”.
4. page 42, line -3: change $Cov[X, Y] = Var[X]$ to $Cov[X, X] = Var[X]$.
5. page 44, just before inequality 4.5: insert the following **Exercise:** (Littlewood’s inequality) Let $\mu_r \equiv E|X|^r$. For $r \geq s \geq t \geq 0$ we have $\mu_r^{s-t} \mu_t^{r-s} \geq \mu_s^{r-t}$.

6. page 45, line -3: change $u^\alpha v^{\alpha-1}$ to $u^\alpha v^{1-\alpha}$ to
7. page 49, (15): change “unifrom” to “uniform”.

Chapter 4.

1. page 55, line -3: change \int to \int_A three times.
2. page 58, line 9: in the definition of A_k , change $\max_{1 \leq k < j} Z_j$ to $\max_{1 \leq j < k} Z_j$; also note that the inclusion of this is crucial to make the A_k 's form a partition of Ω .
3. page 58, line -7: change the inequality sign \leq in the line before (h) to an equality sign; change the first equality sign in (h) to a \leq sign.
4. page 59, (k) and (l): replace by the following:

$$\begin{aligned}
\phi_{ac}(A) &= \int_A Z_0 d\mu = \int_{A\Omega^-} Z_0 d\mu \quad \text{since } \mu(\Omega^+) = 0 \\
&= \phi_{ac}(A\Omega^-) \\
&= \phi_{ac}(A\Omega^- \overline{\Omega}^-) + \phi_{ac}(A\Omega^- \overline{\Omega}^+) \\
&= \phi_{ac}(A\Omega^- \overline{\Omega}^-) + \int_{A\Omega^- \overline{\Omega}^+} Z_0 d\mu \\
&= \phi_{ac}(A\Omega^- \overline{\Omega}^-) \quad \text{since } \mu(\overline{\Omega}^+) = 0 \\
&= \phi(A\Omega^- \overline{\Omega}^-) - \phi_s(A\Omega^- \overline{\Omega}^-) \\
&= \phi(A\Omega^- \overline{\Omega}^-) \quad \text{since } \phi_s(\Omega^-) = 0 \\
&= \overline{\phi}(A\Omega^- \overline{\Omega}^-) \quad \text{by hypothesis} \\
&= \overline{\phi}_{ac}(A) \quad \text{by repeating the above steps in reverse}
\end{aligned}$$

5. page 60, line 3: change “for some” to “for”.
6. page 61, (5): change “a.e. ν ” to “a.e. μ ”.
7. page 61, (6): change “a.e. μ ” to “a.e. ν ”.
8. page 61, Exercise 2.2: I would advocate replacing this exercise by the following: “Let P_{μ, σ^2} denote the $N(\mu, \sigma^2)$ distribution on R .
(a) Show that $P_{\mu, 1} \ll P_{0, 1}$ and compute $dP_{\mu, 1}/dP_{0, 1}$.
(b) Show that $P_{0, \sigma^2} \ll P_{0, 1}$ and compute $dP_{0, \sigma^2}/dP_{0, 1}$.”

9. page 61, Exercise 2.3: Because P is not absolutely continuous with respect to either Lebesgue measure or counting measure, I would advocate changing this to read as follows:
 - (a) Let μ be Lebesgue measure on R . Find the Lebesgue decomposition of P with respect to μ , $P = P_{ac} + P_s$.
 - (b) Repeat (a) with μ being counting measure on $\{0, 1, 2, \dots\}$.
10. page 62, line 7: change x to x (after “differentiable at”).

Chapter 5.

1. page 71, (1): change the last A' to A'_i .
2. page 71, (2) +1: change “T he” to “The”.
3. page 72, line 5: change “Carethéodory” to “Carathéodory”.
4. page 73, line 12: change “monotonicy” to “monotonically” twice.
5. page 74, line 9: change “interated” to “iterated”.

Chapter 7.

1. page 98, line -15: change $\mu_c(x, y]$ to $\mu_{ac}(x, y]$.
2. page 99, line 5: change $0 < t <$ to $0 \leq t \leq 1$;
3. page 99, lines 8 and 19: change $\cup_{0 < t < 1} U_t$ to $\cup_{0 \leq t \leq 1} U_t$.
4. page 112, line -3: change “materail” to “material”.

Chapter 8.

1. page 133, lines 5-6: change “occurence” to “occurrence” four times.
2. page 133, line 14: change $A_i \in \mathcal{A}$ to $A_i \in \mathcal{A}_i$.
3. page 133, line -5: add after “if and only if”, “... , for each $k = 1, \dots, n$,”
4. page 134, line 3: change “then” to “than”.
5. page 154, (19): change $\rightarrow_{\mathcal{L}_r}$ and $a.s.$ to $\rightarrow_{\mathcal{L}_1}$ and $a.s.$:

Chapter 9.

1. page 156, formula (6): change $\binom{N}{k}$ to $\binom{N}{n}$.
2. page 156, line -1: change Hypergeometric($m+n, k, m$) to Hypergeometric($m+n, m, k$).
3. page 163, formula (9): on the right side replace $\int_0^\infty y f_X(yz) dy$ by $\int_0^\infty y f_X(yz) f_Y(y) dy$.
4. page 166, line -4: change “Since $Y^{(1)} = Z^{(1)} + \dots$ ” to $Y^{(1)} = Z^{(1)} + \mu^{(1)} + \dots$

Chapter 10.

1. page 173, line 4: change “test” to “text”.
2. page 181, Proof of Inequality 3.4: Change the first sentence of the proof to read as follows: “Let κ denote the first $k \leq n$ for which $S_k \geq \lambda$. Thus $[\kappa = k] = [S_1 < \lambda, \dots, S_{k-1} < \lambda, S_k \geq \lambda]$.”
3. page 181, (d): to make the proof work, change the definition of a to $a \equiv \min_{1 \leq k \leq n} P(S_n - S_k \geq -(1-c)\lambda)$.
4. page 181, Proof of Inequality 3.4: Here’s my preferred proof.

Proof. Let κ denote the first $k \leq n$ for which $|S_k| \geq \lambda$. Thus $[\kappa = k] = [|S_1| < \lambda, \dots, |S_{k-1}| < \lambda, |S_k| \geq \lambda]$, and $\sum_{k=1}^n [\kappa = k] = [\max_{k \leq n} |S_k| \geq \lambda]$. Then, with

$$a \equiv \min_{1 \leq k \leq n} P(|S_n - S_k| < (1-c)\lambda),$$

$$\begin{aligned} aP(\max_{1 \leq k \leq n} |S_k| \geq \lambda) &\leq \sum_{k=1}^n P(|S_n - S_k| < (1-c)\lambda)P(\kappa = k) \\ &= \sum_{k=1}^n P([\kappa = k] \cap [|S_n - S_k| < (1-c)\lambda]) \quad \text{by independence} \\ &= \sum_{k=1}^n P([\kappa = k] \cap [|S_k| \geq \lambda] \cap [|S_n - S_k| < (1-c)\lambda]) \end{aligned}$$

$$\begin{aligned}
&\leq \sum_{k=1}^n P([\kappa = k] \cap [|S_n| \geq c\lambda]) \\
&= P(|S_n| \geq c\lambda).
\end{aligned}$$

5. page 182, line 3: change X_x at the end of the line to X_k .
6. page 183, line 1: change “unifrom” to “uniform”.
7. page 185, (g): this should read as $\sum_{n=1}^{\infty} \sigma_n^2/n^2 = \sum_{n=1}^{\infty} \text{Var}[Y_n]/n^2 < \infty$.
8. page 187, (b): change this line to read as follows:

$$\frac{2}{n\epsilon^2} \int_0^n \tau(x)dx \leq \frac{2}{n\epsilon^2} \left\{ \int_0^M \tau(x)dx + 4^{-1}\epsilon^3(n - M) \right\} \leq \frac{M^2}{n\epsilon^2} + \frac{\epsilon}{2} \leq \epsilon$$

(by using $\tau(x) \leq x$ for the first term) for all $n \geq 2M^2/\epsilon^3$.

9. page 190, (4): change $\|\mathbb{G}_n - I\|$ to $\|\mathbb{G}_n - I\|$.
10. page 190, (a): change $Ef(T)$ to $Ef(T/n)$.
11. page 191, (b), proof for Example 5.2: change $n^{-1} \sum_1^n \eta_{nk}$ to $n^{-1} \sum_{j=1}^n \eta_{jk}$.
12. page 200, line 2: change “Winter” to “Wintner”. (see e.g. Dudley (1989), page 380).
13. page 200, line 7: change (b) to (c) and move “(c) Conversely, if” to the next line.
14. page 201, (g): replace the ratio $b_{n_k}/b_{n_{k-1}}$ by $b_{n_{k-1}}/b_{n_k}$.
15. page 202, line 9: change “Winter” to “Wintner”.
16. page 206, exercise 9.3: change $\sum_{k=1}^{\infty} X_k \rightarrow_{a.s.}$ to $\sum_{k=1}^n X_k \rightarrow_{a.s.}$

17. page 208, (g): From (g) to the end of the proof there are several minor difficulties. In replace (g) and the next line, by

$$\begin{aligned} \text{Var}(S) &= \frac{1}{2}E(S - S')^2 = \frac{1}{2}E\{\lim(S_n - S'_n)^2\} = \frac{1}{2}E\{\underline{\lim}(S_n - S'_n)^2\} \\ &\leq \frac{1}{2}\underline{\lim}E\{(S_n - S'_n)^2\} \quad \text{by Fatou's lemma} \\ &\leq \underline{\lim}\text{Var}(S_n) \end{aligned}$$

In the line below (i), to show that $S_n \rightarrow_2 S$, argue that

$$\begin{aligned} E(S_m - S_n)^2 &= \text{Var}(S_m - S_n) + \{E(S_m - S_n)\}^2 \\ &= \sum_{k=n+1}^m \sigma_k^2 + \left\{ \sum_{k=n+1}^m \mu_k \right\}^2 \rightarrow 0 \end{aligned}$$

as $m, n \rightarrow \infty$, so that $\{S_n\}$ is Cauchy in L_2 , and hence $S_n \rightarrow_2 S$. Then $E(S_n) = \sum_{k=1}^n \mu_k \rightarrow \mu$ by uniform integrability.

Chapter 11.

1. page 217, line 2: Change “Distribtuion” to “Distribution”.
2. page 218, (8): replace $\|h'\|$ by $2\|h'\|$.
3. page 218, -14: replace $x \leq 0$ by $x \leq z$; and replace $x \geq 0$ by $x \geq z$.
4. page 221, (22): delete everything after $M_n \rightarrow_p 0$; the first inequality seems to be false, and the rest of the line depends on this.
5. page 221, (23): at the end of this display, change 3ϵ to 5ϵ .
6. page 222, line 6: replace “Bolthausen’s” by “Stein’s”.
7. page 234, line -16: in Exercise 11.5.1, change “Unifrom” to “Uniform”.
8. page 244, lines -2&-1: Remark 8.1 seems out of place.

Chapter 12.

1. page 254, line 11: change “then” to “than”.

2. page 258, (8): change A_{T_0} to \mathcal{A}_{T_0} .
3. page 258, -10: change “continuos” to “continuous”.
4. page 264, (a)-1 to (f): I'd suggest changing t to x throughout this proof (for clarity)
5. page 265, (1): change $(a, b)^c$ to $(-a, b)^c$.
6. page 265, proof of theorem 7.1: For clarity, start the proof with: “First the proof of (7.1): we temporarily set $\tau \equiv \tau_a$.”
7. page 266, (8) and (a)-(b): It seems to me that $\exp(\theta SS(t) - \theta^2 t/2)$ is a martingale; thus I'd replace “ $V_\theta(t) = \exp(-[\theta SS(t) - \theta^2 t/2])$ ” by “ $V_\theta(t) = \exp(\theta SS(t) - \theta^2 t/2)$ ”. Also note that the equality sign in (b) of the proof is incorrect since

$$E \exp(-\theta N(0, t-s)) = E \exp(i(i\theta)N(0, t-s)) = \exp(-(i\theta)^2(t-s)/2) = \exp(\theta^2(t-s)/2)$$
8. page 268, (1): in the first summation, change the index from k to i .
9. page 268, (4): change $F(0, \sigma^2)$ to $F(0, 1)$.
10. page 268, (4)+1: change \mathbf{S} to \mathbf{S}_n .
11. page 268, (8)+2: change $F(0, \sigma^2)$ to $F(0, 1)$ in the statement of the theorem.
12. page 269, (b)+1: change “(10)” to “(9)”.
13. page 269, (e)+1: change “(10)” to “(9)”.
14. page 269, (11): change $\|S_n - S\|_\infty$ to $\rho_\infty(S_n, S)$.
15. page 270, (8): change $g(\mathcal{S}_n)$ and $g(\mathcal{S})$ to $g(\mathbf{S}_n)$ and $g(\mathbf{S})$.
16. page 271, (2): change $\text{Log}k \equiv 1 \wedge (\log k)$ to $\text{Log}k \equiv 1 \vee (\log k)$.
17. page 271, (a)+1: after “monotone inequality” insert “Inequality 10.11.1”.

18. page 273, Exercise 9.1: change the hint to “... where

$$SS_n - 1 \equiv \sum_{k=1}^n \{SS^2(t_{n,k-1}, t_{n,k}] - (t_{n,k} - t_{n,k-1})\}."$$

19. page 277, (40): To have notation match correctly, this line should be replaced by

$$\eta_{n+1,k} \equiv k + X_{n+1,1} + \cdots + X_{n+1,k} \quad \text{and} \quad \xi_{n:k} \equiv \eta_{n+1,k} / \eta_{n+1,n+1} \quad \text{for } 1 \leq k \leq n+1.$$

20. page 277, (a)-(41) and (43)-1: replace η_k and η_n by $\eta_{n,k}$ and $\eta_{n,n}$ throughout these lines.

21. page 277, (b): replace this line by:

$$\|\mathbb{S}_n(I_n) - \mathbb{S}\| \leq \|\mathbb{S}_n(I_n) - \mathbb{S}(I_n)\| + \|\mathbb{S}(I_n) - \mathbb{S}(I)\| \leq \|\mathbb{S}_n - \mathbb{S}\| + \|\mathbb{S}(I_n) - \mathbb{S}(I)\| \rightarrow_p 0,$$

22. page 278, (44): change $\max_{1 \leq k \leq n+1}$ to $\max_{1 \leq i \leq n+1}$.

Chapter 13.

1. page 287, line 16: change “complex plain” to “complex plane”.
2. page 294, theorem 3.3: in the statement, “.” seems to mean two different things on the two sides of the equality. Thus it is probably better to write it out to make the statement clear: something like $\phi_{\underline{X}}(\underline{t}) = \phi_{\underline{X}}(X_1, \dots, X_k) = \prod_{j=1}^k \phi_{X_j}(t_j)$ would seem to be better.
3. page 295, (a) of the proof: the organization here suggests that $|t| \leq 2|t/2|^\delta$ for both $|t/2| \leq 1$ and for $|t/2| \geq 1$; but the second of these inequalities is clearly false. Rather the bound follows by reasoning that the far left side is bounded by 2, which is bounded in turn by $2|t/2|^\delta$ on the set $|t/2| \geq 1$.
4. page 295, (5), line -1: change $\sum_{k=1}^m$ to $\sum_{k=0}^m$.

5. page 296, lemma 4.3: this was originally stated in terms of three hypotheses:

(i) $\theta_n = \sum_{k=1}^n \theta_{nk} \rightarrow \theta$ as $n \rightarrow \infty$.

(ii) $\delta_n \equiv \max_{1 \leq k \leq n} |\theta_{nk}| \rightarrow 0$.

(iii) $M_n \equiv \sum_{k=1}^n |\theta_{nk}| \leq (\text{some } M) < \infty$.

Since the first line of the proof requires $\delta_n \rightarrow 0$, it would seem better to keep (i) - (iii) as the hypotheses.

6. page 296, lemma 4.4: Another proof would invoke the formula

$$\prod_{k=1}^n z_k - \prod_{k=1}^n w_k = \sum_{k=1}^n \left(\prod_{j=1}^{k-1} z_j \right) (z_k - w_k) \left(\prod_{l=k+1}^n w_l \right).$$

7. page 296, line -3: change “establishs” to “establishes”.

8. page 297, Exercise 4.1: The lower bound part of this exercise is false. I do not yet have a proof of the upper bound part.

9. page 297, Exercise 4.1: A possible addition here would be as follows. Show that with

$$\Delta_n \equiv \sup_{x \geq 0} |(1 - x/n)^n 1_{[0,n]}(x) - e^{-x}|$$

we have $2e^{-2} \leq n\Delta_n \leq (2 + n^{-1})e^{-2}$ for all $n \geq 1$. (This is from Hall and Wellner (1979), *Statistica Neerl.* 33, 151-154.)

Chapter 14.

1. page 308, (a): change $\prod_{j=1}^n$ to $\prod_{k=1}^n$.
2. page 308, (d) and (e): change $\phi_{\sqrt{n}(X_{nk}-\mu)}$ to $\phi_{(X_{nk}-\mu)/\sqrt{n}}$ twice.
3. page 309, line -10; (a) of the proof of Theorem 1.2: delete the $/n$ near the end of this line. It shouldn't be there.
4. page 309, after Theorem 14.1.2.: Possible additional exercise: Suppose that X_{n1}, \dots, X_{nn} are independent random variables with $X_{nk} \sim \text{Bernoulli}(p_{nk})$, and let $Y_{nk} \sim \text{Poisson}(p_{nk})$ for $k = 1, \dots, n$. Let P_n be

the distribution of $X_n \equiv \sum_{k=1}^n X_{nk}$ and let Q_n be the distribution of $Y_n \equiv \sum_{k=1}^n Y_{nk}$. Show that

$$d_{TV}(P_n, Q_n) \equiv \sup\{|P_n(A) - Q_n(A)| : A \in \mathcal{B}\} \leq \sum_{k=1}^n p_{nk}^2.$$

Note that if $p_{nk} = \lambda_k/n$ for $k = 1, \dots, n$, then the bound becomes $\bar{\lambda}/n$.

Remark: Another bound due to Le Cam (1960) and Kerstan (1964) as as follows:

$$d_{TV}(P_n, Q_n) \leq 1.05 \frac{1}{\sum_{k=1}^n p_{nk}} \sum_{k=1}^n p_{nk}^2;$$

this is often better than the preceding bound when $\sum_{k=1}^n p_{nk}$ is large. See Barbour, Holst, and Janson (1992) page 4 for a nice discussion of this.

[Hint: Construct X_n and Y_n on a common probability space as follows: Let $T_{nk} \sim \text{Poisson}(p_{nk})$, $k = 1, \dots, n$ and $Z_{nk} \sim \text{Bernoulli}(1 - (1 - p_{nk})e^{p_{nk}})$, $k = 1, \dots, n$ all be independent, and define

$$X_{nk} = 1_{[T_{nk} \geq 1]} + 1_{[T_{nk}=0]}1_{[Z_{nk}=1]}.$$

Set $X_n \equiv \sum_{k=1}^n X_{nk}$ and $Y_n = \sum_{k=1}^n T_{nk}$. Check that $X_{nk} \sim \text{Bernoulli}(p_{nk})$,

$$\begin{aligned} P(T_{nk} = 0, X_{nk} = 1) &= e^{-p_{nk}} - (1 - p_{nk}), \\ P(T_{nk} \geq 1, X_{nk} = 0) &= 0, \\ P(T_{nk} \geq 2) &= 1 - e^{-p_{nk}} - p_{nk}e^{-p_{nk}}. \end{aligned}$$

Show that

$$d_{TV}(P_n, Q_n) \leq P(X_n \neq Y_n) \leq \sum_{k=1}^n P(X_{nk} \neq T_{nk}) \leq \sum_{k=1}^n p_{nk}^2.$$

5. page 310, -8: change “classic” to “classical”?
6. page 318, -3: change “Suppose $\sum_{k=1}^n |X_{nk}|^r / \sigma_n^{r/2} \rightarrow 0 \dots$ ” to “Suppose $\sum_{k=1}^n E|X_{nk}|^r / \sigma_n^r \rightarrow 0 \dots$ ”? Also delete the μ_{nk} once (or insert it twice).

7. page 324, (2), line 6: change $d_{TV}(P_n, P)$ to $d_{TV}(P_n, P)$.

Chapter 15.

1. page 336, line 8: change “uiniformly” to “uniformly”.
2. page 347, (9): it would seem that \mathcal{U}_α here should be $\mathcal{U}_{-\alpha}$?
3. page 347, -3: in part (a) of Exercise 4.3, it seems that the factor of $|x|$ in the numerator should be deleted (and the constant checked?). With

$$f(x) = \frac{\log |x|}{2|x|^2} 1_{[1, \infty)}(|x|),$$

this example works for exercise 4.2(c).

Chapter 17.

1. page 390, (q): for the fourth term inside $\{\dots\}$, I get $(1.15)(32)(N - 1)^{3/2}/N^2$.
2. page 390, (13)+1: change $n \geq 4$ to $N \geq 4$.
3. page 391, (19): replace the \rightarrow here by \sim . (This one rubs wrong ... since I have been trying to get students to avoid writing a convergence arrow with the quantity converging appearing to the right of it during both the 580's and 520's!)
4. page 391, (23): replace T_N by $\sqrt{N-1}(T_N/N)$.
5. page 392, (31): The first inequality sign in this line (from the left) can be replaced by equality.
6. page 392, (32): The last inequality sign in this line can be replaced by equality.

Chapter 18.

1. page 408, line -9: change X_{nj} to X_{nj} .
2. page 431, line -7: change “trun” to “turn”.
3. page 448, line -3: change “Continuoius” to “Continuous”.

Chapter Index.

1. page 466, line -4: change “matingale” to “martingale”.