by (4.36). By (4.2) and (4.3) one has that

$$\sum_{L=1}^{\infty} |p^{(L)}(e^{i\lambda})|^2 u_L(e^{i\lambda}) = \begin{cases} 1 \text{ for } \lambda \in [-\pi, \pi] - \{0\} \\ 0 \text{ for } \lambda = 0 \end{cases}.$$

Hence for each $n \in \mathbb{Z}$, by (4.35), (4.33) and (4.34),

$$EX_0X_n = \sum_{L=1}^{\infty} \sum_{M=1}^{\infty} EX_0^{(L)}X_n^{(M)} = \sum_{L=1}^{\infty} EX_0^{(L)}X_n^{(L)} + \sum_{L \neq M} 0$$

$$= \sum_{L=1}^{\infty} \int_{-\pi}^{\pi} e^{in\lambda} f(X^{(L)}, e^{i\lambda})\, d\lambda$$

$$= \int_{-\pi}^{\pi} e^{in\lambda} \sum_{L=1}^{\infty} f(X^{(L)}, e^{i\lambda})\, d\lambda$$

$$= \int_{-\pi}^{\pi} e^{in\lambda} \frac{1}{2\pi}\, d\lambda$$

$$= \begin{cases} 1 \text{ if } n = 0 \\ 0 \text{ if } n \neq 0 \end{cases}.$$

Thus (1.6) holds. The key idea for this argument for (1.6), arranging things so that $\sum_{L=1}^{\infty} f(X^{(L)}, e^{i\lambda})$ is (equivalent to) a constant, came from Herrndorf [11].

Proof of (1.7) Our first task is to show that

$$\forall L \geqq 1,\ \beta(X^{(L)}, 1) \leqq 3 \cdot 2^{-L} \tag{4.38}$$

and

$$\forall L \geqq 1,\ \beta(X^{(L)}, n) \to 0 \quad \text{as} \quad n \to \infty. \tag{4.39}$$

Let $L \geqq 1$ be arbitrary but fixed. By (4.25) and (4.7)/(4.19)/Lemma 3.3(iv), we have that $\beta(W^{(L)}, 1) \leqq 2^{-L}$ and $\beta(W^{(L)}, n) \to 0$ as $n \to \infty$. By (4.27) and Lemma 2.4(iv) and (vi),

$$\beta(Y^{(L)}, 1) \leqq 2^{-L} \quad \text{and} \quad \beta(Y^{(L)}, n) \to 0 \quad \text{as} \quad n \to \infty. \tag{4.40}$$

has that

$$\begin{cases} 1 \text{ for } \lambda \in [-\pi, \pi] - \{0\} \\ 0 \text{ for } \lambda = 0 \end{cases}$$

(4.33) and (4.34),

$$Y_n^{(M)} = \sum_{L=1}^{\infty} EX_0^{(L)} X_n^{(L)} + \sum_{L \neq M} 0$$

$$), e^{i\lambda})\, d\lambda$$

$$^{L)}, e^{i\lambda})\, d\lambda$$

r this argument for (1.6), arranging s (equivalent to) a constant, came

s to show that

$$^{(L)}, 1) \leqq 3 \cdot 2^{-L} \tag{4.38}$$

$$n) \to 0 \quad \text{as} \quad n \to \infty. \tag{4.39}$$

. By (4.25) and (4.7)/(4.19)/Lemma 2^{-L} and $\beta(W^{(L)}, n) \to 0$ as $n \to \infty$. By i),

$$\beta(Y^{(L)}, n) \to 0 \quad \text{as} \quad n \to \infty. \tag{4.40}$$

Also by (4.32),

$$\sigma(X_k^{(L)}, k \leqq 0) \subset \sigma(Y_k^{(L)}, k \leqq 0) \tag{4.41}$$

and

$$\sigma(X_k^{(L)}, k \geqq 1) \subset \sigma(Y_k^{(L)}, k \geqq 1 - J_L). \tag{4.42}$$

If $J_L = 0$, then (4.38) and (4.39) follow from (4.40), (4.41) and (4.42). Now assume instead that $J_L \geqq 1$. Then $L \geqq 2$ by (4.5). Now

$$P(Y_k^{(L)} \neq 0 \text{ for some } k \in \{1 - J_L, 2 - J_L, \ldots, 0\})$$

$$\leqq J_L \cdot P(Y_0^{(L)} \neq 0) \leqq J_L \cdot 1/N_L \leqq 1/N_L^{1/2} \leqq 2^{-L}$$

by (4.28), (4.17) and (4.14). Hence $\sigma(Y_k^{(L)}, 1 - J_L \leqq k \leqq 0)$ has an atom $\{Y_k^{(L)} = 0 \, \forall \, k = 1 - J_L, 2 - J_L, \ldots, 0\}$ which has probability $\geqq 1 - 2^{-L}$. Hence by (4.40), (4.41), (4.42) and Lemma 2.2 (with $\mathscr{A}, \mathscr{B}, \mathscr{C}$ there being $\sigma(Y_k^{(L)}, k \leqq 0)$, $\sigma(Y_k^{(L)}, 1 - J_L \leqq k \leqq 0)$, and $\sigma(Y_k^{(L)}, k \geqq 1)$ here), one has

$$\beta(X^{(L)}, 1) \leqq 2 \cdot 2^{-L} + \beta(Y^{(L)}, 1) \leqq 3 \cdot 2^{-L}.$$

Thus (4.38) holds. Also, for each $n > J_L$, $\beta(X^{(L)}, n) \leqq \beta(Y^{(L)}, n - J_L)$ by (4.32). Hence (4.39) follows from (4.40). This completes the proof of (4.38) and (4.39).

To complete the proof of (1.7), note that for each $L \geqq 1$ one has the trivial fact

$$\forall n \geqq 1, \; \beta(X^{(L)}, n) \leqq \beta(X^{(L)}, 1).$$

Also, $\sum_{L=1}^{\infty} \beta(X^{(L)}, 1) < \infty$ by (4.38). Hence by (4.39) and dominated convergence, $\sum_{L=1}^{\infty} \beta(X^{(L)}, n) \to 0$ as $n \to \infty$. Now $\beta(X, n) \leqq \sum_{L=1}^{\infty} \beta(X^{(L)}, n)$ for all $n \geqq 1$ by (4.37), (4.35) and Lemma 2.1; and hence (1.7) holds.

Proof of (1.8) By (1.6) and Cauchy's inequality, $n^{-1/2} E|S_n| \leqq n^{-1/2} \|S_n\|_2 = 1$ for all $n \geqq 1$.

The proofs of (1.9) and (1.10) will be facilitated by the following lemma:

LEMMA 4.1 *For each* $L \geqq 2$, *each* $n \in \{N_L^{1/2}, N_L\}$, *one has*

i) $1-2^{-L} \leqq (1/n)E(X_1^{(L)}+\cdots+X_n^{(L)})^2 \leqq 1$ *and*

ii) $(1/n)\,E[(X_1+\cdots+X_n)-(X_1^{(L)}+\cdots+X_n^{(L)})]^2 \leqq 2^{-L}$.

Proof of Lemma 4.1 Let $L \geqq 2$ and $n \in \{N_L^{1/2}, N_L\}$ be arbitrary but fixed.

Note that by (4.37),

$$(X_1+\cdots+X_n)-(X_1^{(L)}+\cdots+X_n^{(L)}) = \sum_{l \neq L} (X_1^{(l)}+\cdots+X_n^{(l)})$$

which is independent of $(X_1^{(L)}+\cdots+X_n^{(L)})$ by (4.35). Hence by (4.33) and (1.6) (already proved above),

$$\frac{1}{n}E[X_1^{(L)}+\cdots+X_n^{(L)}]^2+\frac{1}{n}E[(X_1+\cdots+X_n) -(X_1^{(L)}+\cdots+X_n^{(L)})]^2 = 1. \tag{4.43}$$

By (4.33) and (3.1), and a well known equation (see e.g. [15, Theorem 18.2.1]),

$$\begin{aligned}\frac{1}{n}E[X_1^{(L)}+\cdots+X_n^{(L)}]^2 &= \int_{-\pi}^{\pi} F_n(e^{i\lambda}) f(X^{(L)}, e^{i\lambda})\,d\lambda \\ &= \int_{-\pi}^{\pi} F_n(e^{i\lambda}) \cdot \frac{1}{2\pi}|p^{(L)}(e^{i\lambda})|^2 u_L(e^{i\lambda})\,d\lambda\end{aligned} \tag{4.44}$$

where the second equality follows from (4.34). By (4.21)/(4.22)/ Lemma 3.4(ii),

$$\begin{aligned}&\int_{-\pi}^{\pi} F_n(e^{i\lambda}) \cdot \frac{1}{2\pi}|p^{(L)}(e^{i\lambda})|^2 u_L(e^{i\lambda})\,d\lambda \\ &\geqq \int_{-\pi}^{\pi} F_n(e^{i\lambda}) \cdot \frac{1}{2\pi}|p^{(L)}(e^{i\lambda})|^2\,d\lambda - \varepsilon_L\end{aligned}$$

each $n\in\{N_L^{1/2},N_L\}$, *one has*

$\cdot+X_n^{(L)})^2\leqq 1$ *and*

$X_1^{(L)}+\cdots+X_n^{(L)})]^2\leqq 2^{-L}$.

$\geqq 2$ and $n\in\{N_L^{1/2},N_L\}$ be arbitrary but

$\cdots+X_n^{(L)})=\sum_{l\neq L}(X_1^{(l)}+\cdots+X_n^{(l)})$

$+\cdots+X_n^{(L)})$ by (4.35). Hence by (4.33) ve),

$_n^{(L)}]^2+\frac{1}{n}E[(X_1+\cdots+X_n)$

$X_n^{(L)})]^2=1. \quad (4.43)$

a well known equation (see e.g. [15,

$=\int_{-\pi}^{\pi}F_n(e^{i\lambda})f(X^{(L)},e^{i\lambda})\,d\lambda$

$$=\int_{-\pi}^{\pi}F_n(e^{i\lambda})\cdot\frac{1}{2\pi}|p^{(L)}(e^{i\lambda})|^2u_L(e^{i\lambda})\,d\lambda \quad (4.44)$$

follows from (4.34). By (4.21)/(4.22)/

$|p^{(L)}(e^{i\lambda})|^2u_L(e^{i\lambda})\,d\lambda$

$\lambda)\cdot\frac{1}{2\pi}|p^{(L)}(e^{i\lambda})|^2\,d\lambda-\varepsilon_L$

$$\geqq 1-2^{-(L+1/2)}-3\varepsilon_L$$

$$\geqq 1-2^{-L} \quad (4.45)$$

where the last two inequalities come from (4.18) and (4.1). Since $(1/n)E[X_1^{(L)}+\cdots+X_n^{(L)}]^2\leqq 1$ trivially by (4.43), Lemma 4.1(i) now follows from (4.44) and (4.45). Lemma 4.1(ii) follows from (4.43) and Lemma 4.1(i). This completes the proof of Lemma 4.1.

Now we shall prove (1.10) first, and then use it in the proof of (1.9).

Proof of (1.10) For each $L=1,2,3,\ldots$, each $x\in\mathbb{R}$, by (4.27) and (4.26),

$$\begin{aligned}
&P(Y_1^{(L)}+\cdots+Y_{N(L)}^{(L)}\leqq x)\\
&=\sum_{l=0}^{N(L)}P(Y_1^{(L)}+\cdots+Y_{N(L)}^{(L)}\leqq x\,|\,V_1^{(L)}+\cdots+V_{N(L)}^{(L)}=l)\\
&\qquad\times P(V_1^{(L)}+\cdots+V_{N(L)}^{(L)}=l)\\
&=I_{[0,\infty)}(x)\cdot P(V_1^{(L)}+\cdots+V_{N(L)}^{(L)}=0)\\
&\qquad+\sum_{l=1}^{N(L)}P(W_1^{(L)}+\cdots+W_l^{(L)}\leqq x\,|\,V_1^{(L)}+\cdots+V_{N(L)}^{(L)}=l)\\
&\qquad\times P(V_1^{(L)}+\cdots+V_{N(L)}^{(L)}=l)\\
&=I_{[0,\infty)}(x)\cdot P(V_1^{(L)}+\cdots+V_{N(L)}^{(L)}=0)\\
&\qquad+\sum_{l=1}^{N(L)}P(W_1^{(L)}+\cdots+W_l^{(L)}\leqq x)\cdot P(V_1^{(L)}+\cdots+V_{N(L)}^{(L)}=l). \quad (4.46)
\end{aligned}$$

For each $L\geqq 2$, by (4.19)/(4.25)/Lemma 3.3(iv),

$$|E(W_0^{(L)})^2-1|\leqq 2^{-L}/N_L^2$$

and

$$\forall\, k\neq 0,\ |EW_0^{(L)}W_k^{(L)}|\leqq 2^{-L}/N_L^2.$$

Hence [see (4.25)] for each $L \geqq 2$, each $l = 1, 2, \ldots, N_L$, $W_1^{(L)} + \cdots + W_l^{(L)}$ is a normal r.v. with mean 0 such that $|\mathrm{Var}(W_1^{(L)} + \cdots + W_l^{(L)}) - l| \leqq 2^{-L}$. Hence for each $l = 1, 2, 3, \ldots$ one has that $W_1^{(L)} + \cdots W_l^{(L)} \to N(0, l)$ in distribution as $L \to \infty$. That is, $\forall x \in \mathbb{R}$, $\forall l = 1, 2, 3, \ldots$

$$\lim_{L \to \infty} P(W_1^{(L)} + \cdots + W_l^{(L)} \leqq x) = (2\pi l)^{-1/2} \int_{-\infty}^{x} e^{-u^2/(2l)}\, du. \tag{4.47}$$

For each $L = 1, 2, 3, \ldots$, $V_1^{(L)} + \cdots + V_{N(L)}^{(L)}$ is a binomial r.v. with parameters N_L and N_L^{-1} by (4.24). Hence by Poisson's classic limit theorem,

$$\forall l = 0, 1, 2, \ldots, \lim_{L \to \infty} P(V_1^{(L)} + \cdots + V_{N(L)}^{(L)} = l) = e^{-1}/l! \tag{4.48}$$

By (4.46), (4.47), (4.48) and a careful but simple limiting argument, one has that

$$Y_1^{(L)} + \cdots + Y_{N(L)}^{(L)} \to F \text{ in distribution as } L \to \infty, \tag{4.49}$$

where F is the distribution function defined in (1.10).

Next, for each $L \geqq 2$, by (4.3) (the part "$=0$ for $\lambda = 0$") and (4.11),

$$\left|1 - 2^{-(L+1/2)} - |p^{(L)}(1)|^2\right| \leqq \varepsilon_L.$$

Hence by (4.2), $|p^{(L)}(1)|^2 \to 1$ as $L \to \infty$. Hence $|p^{(L)}(1)| \to 1$ as $L \to \infty$. By (4.12), $p^{(L)}(1) \to 1$ as $L \to \infty$. Hence by (4.49),

$$(p^{(L)}(1)) \cdot (Y_1^{(L)} + \cdots + Y_{N(L)}^{(L)}) \to F \text{ in distribution as } L \to \infty. \tag{4.50}$$

Next, for each $L \geqq 2$, by (4.31) and (4.32),

$$\sum_{k=1}^{N(L)} X_k^{(L)} = \sum_{k=1}^{N(L)} N_L^{1/2} \sum_{j=0}^{J(L)} p_j^{(L)} Y_{k-j}^{(L)}$$

$$= N_L^{1/2} \sum_{j=0}^{J(L)} p_j^{(L)} \sum_{h=1-j}^{N(L)-j} Y_h^{(L)}$$

$$= N_L^{1/2} \sum_{j=0}^{J(L)} p_j^{(L)} \sum_{h=1}^{N(L)} Y_h^{(L)} + Z_L$$

$$= N_L^{1/2} (p^{(L)}(1)) \cdot (Y_1^{(L)} + \cdots + Y_{N(L)}^{(L)}) + Z_L, \tag{4.51}$$

where

$$Z_L := N_L^{1/2} \sum_{j=1}^{J(L)} p_j^{(L)} \left[\sum_{h=1-j}^{0} Y_h^{(L)} - \sum_{h=N(L)-j+1}^{N(L)} Y_h^{(L)} \right]$$

$$:= 0 \text{ if } J_L = 0.$$

For each $L \geqq 2$ satisfying $J_L \geqq 1$, the r.v. Z_L is a linear combination of the r.v.'s $Y_h^{(L)}$, $1 - J_L \leqq h \leqq 0$, $N_L - J_L + 1 \leqq h \leqq N_L$, and hence $P(Z_L \neq 0) \leqq P(Y_h^{(L)} \neq 0$ for some

$$h \in \{1 - J_L, \ldots, 0\} \cup \{N_L - J_L + 1, \ldots, N_L\})$$

$$\leqq 2J_L \cdot P(Y_0^{(L)} \neq 0)$$

$$\leqq 2J_L \cdot 1/N_L$$

$$\leqq 2N_L^{-1/2}$$

by (4.28) and (4.17). (This equation $P(Z_L \neq 0) \leqq 2N_L^{-1/2}$ also holds trivially for any $L \geqq 2$ satisfying $J_L = 0$.) Hence $Z_L \to 0$ in probability as $L \to \infty$. Hence by (4.50) and (4.51),

$$N_L^{-1/2} \sum_{k=1}^{N(L)} X_k^{(L)} \to F \text{ in distribution as } L \to \infty. \tag{4.52}$$

Finally, by Lemma 4.1,

$$N_L^{-1} E[(X_1 + \cdots + X_{N(L)}) - (X_1^{(L)} + \cdots + X_{N(L)}^{(L)})]^2 \to 0 \text{ as } L \to \infty.$$

Hence $N_L^{-1/2}[(\sum_{k=1}^{N(L)} X_k) - (\sum_{k=1}^{N(L)} X_k^{(L)})] \to 0$ in probability as $L \to \infty$. Hence by (4.52), $N_L^{-1/2} \sum_{k=1}^{N(L)} X_k \to F$ in distribution as $L \to \infty$. Thus (1.10) holds [by (4.15)].

Proof of (1.9) Referring to (4.16), for each $L \geqq 2$ define the positive integer $M_L := N_L^{1/2}$.

We need one trivial fact: For any r.v. Z, one has $Z = Z \cdot I(Z \neq 0)$ and hence

$$E|Z| \leqq [EZ^2 EI^2(Z \neq 0)]^{1/2} = [EZ^2 \cdot P(Z \neq 0)]^{1/2}.$$

For each $L \geqq 2$,

$$\begin{aligned}
&E[M_L^{-1/2}|X_1^{(L)} + \cdots + X_{M(L)}^{(L)}|] \\
&\quad \leqq [P(X_1^{(L)} + \cdots + X_{M(L)}^{(L)} \neq 0)]^{1/2} \\
&\qquad \times M_L^{-1/2}[E(X_1^{(L)} + \cdots + X_{M(L)}^{(L)})^2]^{1/2} \\
&\quad \leqq [P(X_1^{(L)} + \cdots + X_{M(L)}^{(L)} \neq 0)]^{1/2} \cdot 1 \\
&\quad \leqq [P(X_k^{(L)} \neq 0 \text{ for some } k = 1, \ldots, M_L)]^{1/2} \\
&\quad \leqq [P(Y_k^{(L)} \neq 0 \text{ for some } k = 1 - J_L, 2 - J_L, \ldots, M_L)]^{1/2} \\
&\quad \leqq [(M_L + J_L) \cdot P(Y_0^{(L)} \neq 0)]^{1/2} \\
&\quad \leqq [(M_L + J_L)/N_L]^{1/2} \\
&\quad \leqq 2/N_L^{1/4}. \qquad (4.53)
\end{aligned}$$

Here the first inequality comes from the preceding paragraph, the second comes from Lemma 4.1, the third is trivial, the fourth comes from (4.32), the fifth is trivial, the sixth comes from (4.28), and the seventh comes from (4.17).

Also, for each $L \geqq 2$, by Lemma 4.1,

$$\begin{aligned}
&E[M_L^{-1/2}|(X_1 + \cdots + X_{M(L)}) - (X_1^{(L)} + \cdots + X_{M(L)}^{(L)})|] \\
&\quad \leqq M_L^{-1/2} E^{1/2}[(X_1 + \cdots + X_{M(L)}) - (X_1^{(L)} + \cdots + X_{M(L)}^{(L)})]^2 \\
&\quad \leqq 2^{-L/2}.
\end{aligned}$$

Hence for each $L \geqq 2$, by (4.53),

$$E[M_L^{-1/2}|X_1 + \cdots + X_{M(L)}|] \leqq 2^{-L/2} + 2/N_L^{1/4}.$$

any r.v. Z, one has $Z=Z\cdot I(Z\neq 0)$

$)]^{1/2}=[EZ^2\cdot P(Z\neq 0)]^{1/2}$.

$_{L)}|]$

$\neq 0)]^{1/2}$

$\cdot +X^{(L)}_{M(L)})^2]^{1/2}$

$\neq 0)]^{1/2}\cdot 1$

$k=1,\ldots,M_L)]^{1/2}$

$k=1-J_L, 2-J_L,\ldots,M_L)]^{1/2}$

$0)]^{1/2}$

(4.53)

from the preceding paragraph, the the third is trivial, the fourth comes ne sixth comes from (4.28), and the

na 4.1,

$)-(X_1^{(L)}+\cdots+X^{(L)}_{M(L)})|]$

$+X_{M(L)})-(X_1^{(L)}+\cdots+X^{(L)}_{M(L)})]^2$

$X_{M(L)}|]\leqq 2^{-L/2}+2/N_L^{1/4}$.

Since $N_L\to\infty$ as $L\to\infty$ [by (4.14) or (4.15)], one has that

$$E[M_L^{-1/2}|X_1+\cdots+X_{M(L)}|]\to 0 \text{ as } L\to\infty.$$

Consequently, if $\lim_{n\to\infty} n^{-1/2}E|X_1+\cdots+X_n|$ were to exist, it would have to be 0. But then $n^{-1/2}(X_1+\cdots+X_n)$ would converge to 0 in probability as $n\to\infty$, contradicting (1.10) (already proved above). Hence this limit cannot exist, i.e. (1.9) holds. This completes the proof of Theorem 1.

Acknowledgement

The author thanks the referee for his helpful comments.

Note added in proof: The answer to Question 2 for "total attraction" is affirmative, by a simple application of Theorem 1 of Dehling, Denker, and Philipp [8]. The author is indebted to a second referee and to Magda Peligrad, each of whom pointed this out.

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