

## Statistics 522, Problem Set 8

Wellner; 2/22/2017

### Reading:

Wellner Chapter 11, Sections 1-2, pages 1-21.  
Shorack, PFS Sections 9.1-9.3, pages 193-205

**Due:** Wednesday, March 1, 2017.

1. Exercise 11.6.4, page 34, Wellner, Chapter 11 notes.  
Give a direct proof of the equivalence of (i) and (iv) in Proposition 2.2.  
(Hint: consider the functions  $\psi_\epsilon(x) \equiv \psi(x/\epsilon)$  with  $\psi$  as given on page 34.)
2. Exercise 9.2.4, PFS Course Notes, page 199. (Exercise 11.8.4, page 293, PFS 2000.)  
Suppose that  $\log X \sim N(0, 1)$ .  
(i) Show that the density of  $X$  is given by  $f_X(x) = x^{-1} \exp(-(\log x)^2/2)/\sqrt{2\pi}$  for  $x > 0$  (and 0 otherwise).  
(ii) For each  $a \in [-1, 1]$  consider the random variable  $Y_a$  with density

$$f_a(y) = f_X(y)(1 + a \sin(2\pi \log y)) \text{ for } y > 0.$$

Show that  $EX^k = EY_a^k$  for all integers  $k \geq 1$  and  $a \in [-1, 1]$ .

3. Exercise 11.6.7, page 34, Wellner, Chapter 11 notes.
4. Exercise 11.6.9, page 35, Wellner, Chapter 11 notes. Use the Cramér - Wold device to prove the multivariate CLT from the classical (one-dimensional) CLT.
5. **Optional bonus problem 1:** Exercise 11.6.5, page 34, Wellner, Chapter 11 notes.
6. **Optional bonus problem 2:** Exercise 9.1.6, PFS Course Notes, page 197. (Exercise 11.7.6, page 291, PFS 2000).