Statistics 522, Problem Set 7

Wellner; 2/15/2017

Reading:	
Wellner	Chapter 11, Sections 1-2, pages 1-19.
Shorack, PfS	Sections 9.1-9.2, pages 193-200
Due:	Wednesday, February 22, 2017.
Reminder:	Midterm exam, Friday, February 17.

1. Let $\mathbb{S}$ be standard Brownian motion on $[0,\infty)$. For $b > 0$ fixed, let $\tau \equiv \tau_b \equiv \inf\{t > 0 : \mathbb{S}(t) = b\}$. Then τ_b is a stopping time. Use the exponential martingale $Y_r(t) \equiv \exp(r\mathbb{S}(t) - (1/2)r^2 t)$ to give a development for τ_b parallel to that given in class on 13 February for T_b in the context of a simple random walk. That is, use optional sampling of the martingale Y_r to:
   (a) show that $P(\tau_b < \infty) = 1$.
   (b) Show that the Laplace transform of τ_b is given by
   $E\exp(-s\tau_b) = \exp(-\sqrt{2s}b)$ for $s \ge 0$.
   (c) Show that $E(\tau_b) = \infty$.
   (d) The density of τ_b is given by $f_{\tau_b}(t) = (b/t^{3/2})\phi(b/\sqrt{t})1_{(0,\infty)}(t)$ where $\phi(x) = (2\pi)^{-1/2}\exp(-x^2/2)$ is the standard normal density. We will show this via reflection arguments for Brownian motion during Spring quarter. Use this expression for the density to show that $E\tau_b^{1/2} = \infty$ and that $E\tau_b^r < \infty$ for each $0 < r < 1/2$.

2. PfS Course notes, Exercise 9.1.2 page 195. (PfS 2000, Exercise 11.7.2, page 289).

3. PfS Course notes, Exercise 9.1.3 page 195. (PfS 2000, Exercise 11.7.3, page 289).

4. Exercise 11.6.2, page 34, Wellner, Chapter 11, notes.