

Statistics 522, Problem Set 8

Wellner; 3/5/2008

Reading:

Shorack, PFS Sections 12.1-12.3, pages 319 - 329.

Shorack, PFS Sections 13.1 - 13.7, pages 367-390.

Due: Wednesday, March 12, 2008.

Reminder: Final exam, Wednesday, March 19.

1. PFS, Exercise 13.1.4, page 371.

Show that the real part of a characteristic function (or $Re\phi(\cdot)$) is itself a characteristic function.

2. PFS, Exercise 13.4.4, page 381.

Verify that if g and g' are in $\mathcal{L}_1(R, \mathcal{B}, \lambda)$, then $|g(x)| \rightarrow 0$ as $x \rightarrow \infty$.

3. PFS, Exercise 14.1.1, page 392.

For each $n \geq 1$, let X_{n1}, \dots, X_{nn} be i.i.d. with finite mean μ . Use characteristic functions to show the WLLN result that $\bar{X}_n \rightarrow_p \mu$ as $n \rightarrow \infty$. Equivalently, show that

$$\bar{X}_n \rightarrow_d \delta_\mu \equiv \text{the degenerate distribution with mass 1 at } \mu.$$

4. PFS, Exercise 14.1.4, page 393.

(a) Suppose the hypotheses of the classical CLT hold. Show that

$$M_n \equiv \frac{1}{\sqrt{n}} \max_{1 \leq k \leq n} |X_{nk} - \mu| \rightarrow_p 0.$$

(b) Suppose that hypotheses of the classical Poisson Limit Theorem (see Theorem 1.2 on page 393 of PFS) hold. Show that

$$M_n \equiv \max_{1 \leq k \leq n} |X_{nk}| \rightarrow_d \text{Bernoulli}(1 - e^{-\lambda}).$$

5. PFS, Exercise 14.2.9, page 403. The following are equivalent:

(a) $|X_{nk}|$'s are uan, meaning that $\max_{1 \leq k \leq n} P(|X_{nk}| \geq \epsilon) \rightarrow 0$ for all $\epsilon > 0$.

(b) $\max_{1 \leq k \leq n} |\phi_{nk}(t) - 1| \rightarrow 0$ uniformly on every finite interval of t 's.

(c) $\max_{1 \leq k \leq n} \int \alpha dF_{nk}(x) \rightarrow 0$ for $\alpha(x) \equiv x^2 \wedge 1$.

6. **Optional bonus problem:** PfS, Exercise 14.2.10, page 403. Let \bar{Z}_n denote an appropriately normalized \bar{X}_n for a single i.i.d sequence of rv's. Prove or disprove the following statement: $\bar{Z}_n \rightarrow_d N(0, 1)$ if and only if the symmetrized random variables $\bar{Z}_n^s \rightarrow_d N(0, 2)$. (Recall Definition 10.3.1, PfS, page 228.)
7. **Optional bonus problem:** Suppose that X_1, X_2, \dots are i.i.d. with characteristic function $\phi \equiv \phi_X$, and let $N \sim \text{Poisson}(\lambda)$ be independent of all the X_i 's. Show that $S \equiv X_1 + \dots + X_N$ has characteristic function ϕ_S given by

$$\phi_S(t) = \exp(\lambda(\phi_X(t) - 1)).$$

(See PfS, Example 15.1.1, page 430.)