Statistics 522, Problem Set 5
corrected version
Wellner; 2/13/2008, 2/16/2008

Reading:

Wellner	Chapter 11, Sections 3 - 5, pages 19 - 33.
Shorack, PfS	Section 11.7, pages 312 - 318.

Due: Wednesday, February 20, 2008.

Reminder: Midterm exam, February 20.

1. Exercise 11.8.7, page 55, Wellner, Chapter 11 notes. (new numbering; exercise 11.6.7, page 33, old numbering).

2. Exercise 11.8.8, page 55, Wellner, Chapter 11 notes. Corrected version: Let Y be a random vector in $\mathbb{R}^k$ with $\mu = E(Y)$ and $\Sigma = Cov(Y) = E\{(Y-\mu)(Y-\mu)'\}$. Thus we can write $\Sigma = A\Lambda^2 A'$ where A is an orthogonal matrix (so $AA' = I$) and Λ is diagonal with each diagonal entry non-negative. Define $B = A\Lambda$. Let Z be a random vector with independent $N(0,1)$ coordinates; thus $Z \sim N_k(0, I)$. In the following, $|x|$ denotes the Euclidean norm of $x \in \mathbb{R}^k$, $|x| = (\sum_1^k |x_i|^2)^{1/2}$.
(a) Show that $|\mu| \le E\|Y\|$. [Hint: note that $u'Y \le |Y|$ for all unit vectors u, and in particular for $u = \mu/|\mu|$.
(b) Show that $E|BZ|^3 = E|\Lambda Z|^3 \le (\text{trace}(\Sigma))^{3/2} E|Z_1|^3$.
(c) Show that $E|\mu + BZ|^3 \le 8E|Y|^3 + 8(E|Y|^2)^{3/2} E|Z_1|^3$. Can the factor 8 be improved to 4?

3. Exercise 11.8.9, page 55, Wellner, Chapter 11, notes.

4. PfS, exercise 11.8.4, page 317.

5. **Optional bonus problem:**
(a) PfS, exercise 11.8.5, page 317: Show that if $\limsup |\mu_k|^{1/k}/k < \infty$, then at most one distribution function F can possess the moment values $\mu_k = \int x^k dF(x)$.
(b) PfS, exercise 11.8.6, page 318: Show that the standard normal distribution $N(0,1)$ is uniquely determined by its moments.