Statistics 522, Problem Set 6

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Reading: Shorack, PfS, Chapter 13, pages 341 - 357;
Shorack, PfS, Chapter 14, pages 365 - 376

Due: Wednesday, February 18, 2004.

1. PfS, Exercise 4.2, page 354.

2. Find independent random variables X, Y, and Z so that Y and Z do not have the same distribution, but $X+Y$ and $X+Z$ do have the same distribution. Hint: let X have the de la Vallee - Poussin distribution with density $f(x) = [1 - \cos(x)]/\pi x^2$, let $Y =_d X$, and let Z have the same characteristic function as Y, but extended periodically with period 4; identify the probability distribution of Z explicitly. (This is from Feller, vol. II, pages 505-507.)

3. Suppose that a characteristic function ϕ_X has $|\phi_X(t_0)| = 1$ for some non-zero t_0; i.e. $\phi_X(t_0) = \exp(i\theta_0)$ for some real θ_0. Show that $P(X = (\theta_0 + 2k\pi)/t_0 \text{ for some } k \in \mathbb{Z}) = 1$.
Hint: Show that $Re(1 - E\exp(it_0 X - i\theta_0)) = 0$ and examine the function $1 - \cos(t_0 x - \theta_0)$.

4. PfS, Exercise 1.4 (b). Suppose that $X_{n1}, \ldots, X_{nn}$ are independent Bernoulli(λ_{nk}) random variables for which $\sum_1^n \lambda_{nk} \to \lambda$ and $\sum_1^n \lambda_{nk}^2 \to 0$. Show that

$$M_n \equiv \max_{1 \le k \le n} |X_{nk}| \to_d \text{Bernoulli}(1 - e^{-\lambda}) .$$

Note that theorem 1.2, PfS yields

$$\sum_1^n X_{nk} \to_d Y \sim \text{Poisson}(\lambda) .$$