Statistics 522, Problem Set 1

Wellner; 1/7/2004

Reading: Shorack, PfS, Chapter 10, Section 7, pages 235 - 238; Shorack, PfS, Chapter 10, Section 11, pages 250-251

Due: Wednesday, January 14, 2004.

1. Complete the proof of (6) in Inequality 3.4, page 181, PfS, i.e. show how the two one-sided arguments combine to yield the inequality with absolute value signs.

2. Exercise 10.3.3, page 213, PfS.

3. Exercise 10.3.4, page 213, PfS. Hint: Use Jensen's inequality.

4. (A weak law of large numbers under the assumption of uncorrelated summands.) Suppose that $X_1, X_2, \ldots$ are uncorrelated and $E(X_j^2) \le M < \infty$ for all $j \ge 1$. Show that $\overline{X} - E(\overline{X}_n) = (S_n - ES_n)/n \to_p 0$ and $\overline{X}_n - E(\overline{X}_n) \to_2 0$ as $n \to \infty$.

5. (L_1-convergence in the SLLN.) Suppose that $X_1, \ldots, X_n, \ldots$ are i.i.d with $E|X_1| < \infty$. Show that $\overline{X}_n \to_1 \mu \equiv E(X_1)$; i.e. $E|\overline{X}_n - \mu| \to 0$ as $n \to \infty$. [Hint: show that $|\overline{X}_n| \le Y_n$ where Y_n is uniformly integrable, and that this implies the uniform integrability of $\overline{X}_n$. (This is connected with Exercise 10.4.16, page 221.