

Statistics 522
Wellner, 1/11/99

Typos and Suggested Minor Revisions
for *Probability for Statisticians*
by Galen R. Shorack

Chapter 1.

1. page 3: Prop 1.1.(j) equation (4): change “subsets of \mathcal{A} ” to “subsets of Ω ”.
2. page 3: Definition 1.3.(j): Maybe “A subset A of Ω ” or “An arbitrary subset A of Ω ” would be better.
3. page 4: last line of “Motivation 1.2”, just before Example 1.1: change “subets” to “subsets”.
4. page 6: proof of proposition 1.3: in (a), change $\sum_1^n A_n$ to $\sum_1^n A_k$.
5. page 9: Proof, part (b), in the step from 2nd line $\mu^*(TB_n) + \mu^*(TA^c)$ to the 3rd line $\mu^*(TB_nA_1) + \dots$, the \geq can be replaced by $=$. \geq is true and doesn't hurt, but $=$ is also true.
6. page 9: Proof, part (b) just below the preceding item, change $A \in \mathcal{A}$ to $A \in \mathcal{A}^*$.
7. page 14, line -4: change “fine” to “finite”.

Chapter 2.

1. page 22, line -3: change $[-\infty, b) = \cup_{n=1}^{\infty} [-\infty, b + 1/n]$ to $[-\infty, b) = \cup_{n=1}^{\infty} [-\infty, b - 1/n]$.
2. page 24, line -6: change “Suppose $(\Omega, \mathcal{A}, \mu)\dots$ ” to “Suppose $X : (\Omega, \mathcal{A}, \mu)\dots$ ”
3. page 25, Proposition 2.5: change “Suppose that Z is a rv” to “Suppose that Z is a measurable function”.

4. page 25, -14: change “hold” to “holds”.
5. page 25, Exercise 2.2 (Verification criterion): change (18) to: if $V_n \in \mathcal{V}$ satisfy $V_n \uparrow V$, then $V \in \mathcal{V}$; in particular, if $A_n \nearrow A$ with $1_{A_n} \in \mathcal{V}$, then $1_A \in \mathcal{V}$.
6. page 27, exercise 2.3.1: in the first line, change “ (Ω, \mathcal{A}, P) ” to “ $(\Omega, \mathcal{A}, \mu)$ ”.
7. page 28, (5): change $X_n \rightarrow_{a.s.}$ to $X_n \rightarrow_{a.e.}$; change $|X_m - X_n|$ to $|X_m - X|$; also make the second change in (6).
8. page 31, line 1: change “sueful” to “useful”.
9. page 31, line 5: change “probvability” to “probability”.

Chapter 3.

1. page 37, line 4: change (7) to read as follows:

$$\int 2Y = \int \liminf(2Y - Z_n) \leq \liminf \int (2Y - Z_n) = \int 2Y - \limsup \int Z_n,$$

and hence $\limsup \int Z_n d\mu \leq 0$.

2. page 38, line 4: either reverse the inequality in the middle of this display, or (better!) change the display to

$$\begin{aligned} \int (2Y - 0) &= \int \liminf(2Y - Z_n) \leq \liminf \int (2Y - Z_n) \\ &= \liminf \left(\int 2Y - \int Z_n \right) = \int 2Y - \limsup \int Z_n, \end{aligned}$$

and then simplify the conclusion in the next line to just $\limsup \int Z_n d\mu \leq 0$.

3. page 39, line -2: change “Let $\mu_0 = \mu|_{\mathcal{A}}$ ” to “Let $\mu_0 = \mu|_{\mathcal{A}_0}$ ”.
4. page 42, line -3: change $Cov[X, Y] = Var[X]$ to $Cov[X, X] = Var[X]$.
5. page 44, just before inequality 4.5: insert the following **Exercise:** (Littlewood’s inequality) Let $\mu_r \equiv E|X|^r$. For $r \geq s \geq t \geq 0$ we have $\mu_r^{s-t} \mu_t^{r-s} \geq \mu_s^{r-t}$.

6. page 45, line -3: change $u^\alpha v^{\alpha-1}$ to $u^\alpha v^{1-\alpha}$ to
7. page 49, (15): change “unifrom” to “uniform”.

Chapter 4.

1. page 55, line -3: change \int to \int_A three times.
2. page 58, line 9: in the definition of A_k , change $\max_{1 \leq k < j} Z_j$ to $\max_{1 \leq j < k} Z_j$; also note that the inclusion of this is crucial to make the A_k 's form a partition of Ω .
3. page 58, line -7: change the inequality sign \leq in the line before (h) to an equality sign; change the first equality sign in (h) to a \leq sign.
4. page 59, (k) and (l): replace by the following:

$$\begin{aligned}
\phi_{ac}(A) &= \int_A Z_0 d\mu = \int_{A\Omega^-} Z_0 d\mu \quad \text{since } \mu(\Omega^+) = 0 \\
&= \phi_{ac}(A\Omega^-) \\
&= \phi_{ac}(A\Omega^- \overline{\Omega}^-) + \phi_{ac}(A\Omega^- \overline{\Omega}^+) \\
&= \phi_{ac}(A\Omega^- \overline{\Omega}^-) + \int_{A\Omega^- \overline{\Omega}^+} Z_0 d\mu \\
&= \phi_{ac}(A\Omega^- \overline{\Omega}^-) \quad \text{since } \mu(\overline{\Omega}^+) = 0 \\
&= \phi(A\Omega^- \overline{\Omega}^-) - \phi_s(A\Omega^- \overline{\Omega}^-) \\
&= \phi(A\Omega^- \overline{\Omega}^-) \quad \text{since } \phi_s(\Omega^-) = 0 \\
&= \overline{\phi}(A\Omega^- \overline{\Omega}^-) \quad \text{by hypothesis} \\
&= \overline{\phi}_{ac}(A) \quad \text{by repeating the above steps in reverse}
\end{aligned}$$

5. page 60, line 3: change “for some” to “for”.
6. page 61, (5): change “a.e. ν ” to “a.e. μ ”.
7. page 61, (6): change “a.e. μ ” to “a.e. ν ”.
8. page 61, Exercise 2.2: I would advocate replacing this exercise by the following: “Let P_{μ, σ^2} denote the $N(\mu, \sigma^2)$ distribution on R .
(a) Show that $P_{\mu, 1} \ll P_{0, 1}$ and compute $dP_{\mu, 1}/dP_{0, 1}$.
(b) Show that $P_{0, \sigma^2} \ll P_{0, 1}$ and compute $dP_{0, \sigma^2}/dP_{0, 1}$.”

9. page 61, Exercise 2.3: Because P is not absolutely continuous with respect to either Lebesgue measure or counting measure, I would advocate changing this to read as follows:
 - (a) Let μ be Lebesgue measure on R . Find the Lebesgue decomposition of P with respect to μ , $P = P_{ac} + P_s$.
 - (b) Repeat (a) with μ being counting measure on $\{0, 1, 2, \dots\}$.
10. page 62, line 7: change x to x (after “differentiable at”).

Chapter 5.

1. page 71, (1): change the last A' to A'_i .
2. page 71, (2) +1: change “T he” to “The”.
3. page 72, line 5: change “Carethéodory” to “Carathéodory”.
4. page 73, line 12: change “monotonicy” to “monotonically” twice.
5. page 74, line 9: change “interated” to “iterated”.

Chapter 7.

1. page 98, line -15: change $\mu_c(x, y]$ to $\mu_{ac}(x, y]$.
2. page 99, line 5: change $0 < t <$ to $0 \leq t \leq 1$;
3. page 99, lines 8 and 19: change $\cup_{0 < t < 1} U_t$ to $\cup_{0 \leq t \leq 1} U_t$.
4. page 112, line -3: change “materail” to “material”.

Chapter 8.

1. page 133, lines 5-6: change “occurence” to “occurrence” four times.
2. page 133, line 14: change $A_i \in \mathcal{A}$ to $A_i \in \mathcal{A}_i$.
3. page 133, line -5: add after “if and only if”, “... , for each $k = 1, \dots, n$,”
4. page 134, line 3: change “then” to “than”.
5. page 154, (19): change $\rightarrow_{\mathcal{L}_r}$ and $a.s.$ to $\rightarrow_{\mathcal{L}_1}$ and $a.s.$:

Chapter 9.

1. page 156, formula (6): change $\binom{N}{k}$ to $\binom{N}{n}$.
2. page 156, line -1: change Hypergeometric($m+n, k, m$) to Hypergeometric($m+n, m, k$).
3. page 163, formula (9): on the right side replace $\int_0^\infty y f_X(yz) dy$ by $\int_0^\infty y f_X(yz) f_Y(y) dy$.
4. page 166, line -4: change “Since $Y^{(1)} = Z^{(1)} + \dots$ ” to $Y^{(1)} = Z^{(1)} + \mu^{(1)} + \dots$

Chapter 10.

1. page 173, line 4: change “test” to “text”.
2. page 181, Proof of Inequality 3.4: Change the first sentence of the proof to read as follows: “Let κ denote the first $k \leq n$ for which $S_k \geq \lambda$]. Thus $[\kappa = k] = [S_1 < \lambda, \dots, S_{k-1} < \lambda, S_k \geq \lambda]$.”
3. page 181, (d): to make the proof work, change the definition of a to $a \equiv \min_{1 \leq k \leq n} P(S_n - S_k \geq -(1-c)\lambda)$.
4. page 181, Proof of Inequality 3.4: Here’s my preferred proof.

Proof. Let κ denote the first $k \leq n$ for which $|S_k| \geq \lambda$]. Thus $[\kappa = k] = [|S_1| < \lambda, \dots, |S_{k-1}| < \lambda, |S_k| \geq \lambda]$, and $\sum_{k=1}^n [\kappa = k] = [\max_{k \leq n} |S_k| \geq \lambda]$. Then, with

$$a \equiv \min_{1 \leq k \leq n} P(|S_n - S_k| < (1-c)\lambda),$$

$$\begin{aligned} aP(\max_{1 \leq k \leq n} |S_k| \geq \lambda) &\leq \sum_{k=1}^n P(|S_n - S_k| < (1-c)\lambda)P(\kappa = k) \\ &= \sum_{k=1}^n P([\kappa = k] \cap [|S_n - S_k| < (1-c)\lambda]) \quad \text{by independence} \\ &= \sum_{k=1}^n P([\kappa = k] \cap [|S_k| \geq \lambda] \cap [|S_n - S_k| < (1-c)\lambda]) \end{aligned}$$

$$\begin{aligned}
&\leq \sum_{k=1}^n P([\kappa = k] \cap [|S_n| \geq c\lambda]) \\
&= P(|S_n| \geq c\lambda).
\end{aligned}$$

5. page 182, line 3: change X_x at the end of the line to X_k .
6. page 183, line 1: change “unifrom” to “uniform”.
7. page 185, (g): this should read as $\sum_{n=1}^{\infty} \sigma_n^2/n^2 = \sum_{n=1}^{\infty} \text{Var}[Y_n]/n^2 < \infty$.
8. page 187, (b): change this line to read as follows:

$$\frac{2}{n\epsilon^2} \int_0^n \tau(x)dx \leq \frac{2}{n\epsilon^2} \left\{ \int_0^M \tau(x)dx + 4^{-1}\epsilon^3(n - M) \right\} \leq \frac{M^2}{n\epsilon^2} + \frac{\epsilon}{2} \leq \epsilon$$

(by using $\tau(x) \leq x$ for the first term) for all $n \geq 2M^2/\epsilon^3$.

9. page 190, (4): change $\|\mathbb{G}_n - I\|$ to $\|\mathbb{G}_n - I\|$.
10. page 190, (a): change $Ef(T)$ to $Ef(T/n)$.
11. page 191, (b), proof for Example 5.2: change $n^{-1} \sum_1^n \eta_{nk}$ to $n^{-1} \sum_{j=1}^n \eta_{jk}$.
12. page 200, line 2: change “Winter” to “Wintner”. (see e.g. Dudley (1989), page 380).
13. page 200, line 7: change (b) to (c) and move “(c) Conversely, if” to the next line.
14. page 201, (g): replace the ratio $b_{n_k}/b_{n_{k-1}}$ by $b_{n_{k-1}}/b_{n_k}$.
15. page 202, line 9: change “Winter” to “Wintner”.
16. page 206, exercise 9.3: change $\sum_{k=1}^{\infty} X_k \rightarrow_{a.s.}$ to $\sum_{k=1}^n X_k \rightarrow_{a.s.}$

17. page 208, (g): From (g) to the end of the proof there are several minor difficulties. In replace (g) and the next line, by

$$\begin{aligned} \text{Var}(S) &= \frac{1}{2}E(S - S')^2 = \frac{1}{2}E\{\lim(S_n - S'_n)^2\} = \frac{1}{2}E\{\underline{\lim}(S_n - S'_n)^2\} \\ &\leq \frac{1}{2}\underline{\lim}E\{(S_n - S'_n)^2\} \quad \text{by Fatou's lemma} \\ &\leq \underline{\lim}\text{Var}(S_n) \end{aligned}$$

In the line below (i), to show that $S_n \rightarrow_2 S$, argue that

$$\begin{aligned} E(S_m - S_n)^2 &= \text{Var}(S_m - S_n) + \{E(S_m - S_n)\}^2 \\ &= \sum_{k=n+1}^m \sigma_k^2 + \left\{ \sum_{k=n+1}^m \mu_k \right\}^2 \rightarrow 0 \end{aligned}$$

as $m, n \rightarrow \infty$, so that $\{S_n\}$ is Cauchy in L_2 , and hence $S_n \rightarrow_2 S$. Then $E(S_n) = \sum_{k=1}^n \mu_k \rightarrow \mu$ by uniform integrability.

Chapter 11.

1. page 217, line 2: Change “Distribtuion” to “Distribution”.
2. page 218, (8): replace $\|h'\|$ by $2\|h'\|$.
3. page 218, -14: replace $x \leq 0$ by $x \leq z$; and replace $x \geq 0$ by $x \geq z$.
4. page 221, (22): delete everything after $M_n \rightarrow_p 0$; the first inequality seems to be false, and the rest of the line depends on this.
5. page 221, (23): at the end of this display, change 3ϵ to 5ϵ .
6. page 222, line 6: replace “Bolthausen’s” by “Stein’s”.
7. page 234, line -16: in Exercise 11.5.1, change “Unifrom” to “Uniform”.
8. page 244, lines -2&-1: Remark 8.1 seems out of place.

Chapter 18.

1. page 408, line -9: change X_{nj} to X_{nj} .

2. page 431, line -7: change “trun” to “turn”.
3. page 448, line -3: change “Continuoius” to “Continuous”.

Chapter Index.

1. page 466, line -4: change “matingale” to “martingale”.