**Statistics 521, Problem Set 1**

Wellner; 9/27/2019

**Reading:** Shorack, PfS, Chapter 1, pages 1 - 17;
Durrett, Probability, pages 1-13; 394 - 399.

**Due:** Wednesday, October 2, 2019.

1. (a) Suppose that $\{\mathcal{A}_n\}$ is an increasing sequence of algebras, i.e. $\mathcal{A}_n \subset \mathcal{A}_{n+1}$ for all $n \geq 1$. Show that $\cup_{n=1}^{\infty} \mathcal{A}_n$ is an algebra.
(b) Suppose that the $\mathcal{A}_n$ of (a) are $\sigma-$algebras. Show by constructing a counter-example that $\cup_{n=1}^{\infty} \mathcal{A}_n$ need not be a $\sigma-$algebra.

2. Proposition 1.1(b), PfS, page 3: There exists a minimal field, $\sigma-$field, or monotone class generated by (or containing) any specified class $\mathcal{C}$ of subsets of $\Omega$.

3. PfS, Exercise 1.1.1, PfS, page 4: Let $\mathcal{C}_1$ and $\mathcal{C}_2$ denote two collections of subsets of the set $\Omega$. If $\mathcal{C}_1 \subset \sigma[\mathcal{C}_2]$ and $\mathcal{C}_2 \subset \sigma[\mathcal{C}_1]$, then $\sigma[\mathcal{C}_1] = \sigma[\mathcal{C}_2]$.

4. PfS, Exercise 1.1.2, PfS, page 8. We always have $\mu(\liminf A_n) \leq \liminf \mu(A_n)$, while $\limsup \mu(A_n) \leq \mu(\limsup A_n)$ if $\mu(\omega) < \infty$.

5. PfS(2000), Exercise 9.1.4, page 182; or PfS(2012), Exercise A.1.4, page 428. Suppose that $X_n \sim \text{Binomial}(n, p_n)$ where $np_n \to \lambda > 0$. Show that
$$P(X_n = k) \to \frac{\lambda^k}{k!} \exp(-\lambda) = P(Y = k)$$
where $Y \sim \text{Poisson}(\lambda)$; this implies that $X_n \to_d Y$. Can this be strengthened?