

Statistics 521, Problem Set 9

Wellner; 11/21/2012

Reading:

- Shorack, PfS, Chapter 7, pages 123 - 129;
Shorack, PfS, Chapter 8, pages 147 - 174.

Due: Wednesday, November 28, 2012

1. PfS, Exercise 5.3.1, page 94.

Prove that:

- (a) A function $\underline{X} = (X_1, X_2, \dots) : \Omega \rightarrow R^\infty$ is $\mathcal{B}_\infty - \mathcal{A}$ -measurable if and only if each X_n is $\mathcal{B} - \mathcal{A}$ -measurable.
- (b) If $\underline{X} = (X_1, X_2, \dots)$ is $\mathcal{B}^\infty - \mathcal{A}$ -measurable and if (i_1, i_2, \dots) is an arbitrary sequence of integers, then $\underline{Y} \equiv (X_{i_1}, X_{i_2}, \dots)$ is $\mathcal{B}^\infty - \mathcal{A}$ -measurable.

2. PfS, Exercise 7.1.1, page 124:

(a) Show that $P(AB) = P(A)P(B)$ if and only if $\{\emptyset, A, A^c, \Omega\}$ and $\{\emptyset, B, B^c, \Omega\}$ are independent σ -fields. (b) Show that A_1, \dots, A_n are independent if and only if (for each $k = 1, \dots, n$,

$$P(A_{i_1} \dots A_{i_k}) = \prod_{j=1}^k P(A_{i_j}) \quad \text{whenever } 1 \leq i_1 < \dots < i_k \leq n.$$

3. PfS, Exercise 7.1.3, page 126.

If $\mathcal{C}_1, \dots, \mathcal{C}_n$ are independent classes of events and if each \mathcal{C}_i is a $\bar{\pi}$ -system, then $\sigma[\mathcal{C}_1], \dots, \sigma[\mathcal{C}_n]$ are independent σ -fields.

4. Give an example of two collections of sets \mathcal{A}_1 and \mathcal{A}_2 that are independent but the generated σ -fields are not independent.