Statistics 521, Problem Set 4
Wellner; 10/17/2012

Reading:

- Shorack, PfS, Chapter 3, sections 3.1- 3.4, pages 37 - 51.

Reminder: Make-up lectures on Monday 10/29 12:30 - 1:20, CMU 243.
Reminder: Midterm exam: Friday, November 2.

Due: Wednesday, October 24, 2007.

1. PfS, Exercise 2.3.4, page 32: (a) Suppose that $\mu(\Omega) < \infty$ and g is continuous a.e. μ_X (that is, g is continuous except perhaps on a set of μ_X measure 0). Then $X_n \rightarrow_\mu X$ implies that $g(X_n) \rightarrow_\mu g(X)$.
(b) Let g be uniformly continuous on the real line. Then $X_n \rightarrow_\mu X$ implies that $g(X_n) \rightarrow_\mu g(X)$. (Here $\mu(\Omega) = \infty$ is allowed.)

2. PfS, Exercise 3.2.1, page 42: Show that $X \geq 0$ and $\int X d\mu = 0$ implies $\mu([X > 0]) = 0$.

3. PfS, Exercise 3.2.2, page 42: Show that
$$\int_A X d\mu = \begin{cases} = 0, \\ \geq 0, \end{cases} \qquad \text{for all } A \in \mathcal{A} \ \text{ implies } \ X = \begin{cases} = 0 & \text{a.e.,} \\ \geq 0 & \text{a.e.} \end{cases}$$

4. PfS, Exercise 3.2.4, page 43. Let $Y \equiv g(X)$ in the context of Theorem 3.2.6 (the "Theorem of the unconscious statistician"). Show that the second equality holds in:
$$\int_{X^{-1}(g^{-1}(B))} g(X(\omega)) d\mu(\omega) = \int_{g^{-1}(B)} g(x) d\mu_X(x) = \int_B y d\mu_Y(y) \quad \text{for } B \in \overline{\mathcal{B}}$$
where μ_Y is the induced measure of Y on $(\overline{R}, \overline{\mathcal{B}})$.

5. (i) PfS, Exercise 3.3, page 45, part (a).
(ii) Suppose that μ is Lebesgue measure on the unit interval $[0,1]$ and that $(a,b) = (0,1)$ in Exercise 3.3. If $X(t,\omega) = 1_{[\omega \leq t]}$, then for each t, $(\partial/\partial t) X(t,\omega) = 0$ almost everywhere. But $\int X(t,\omega) d\mu(\omega)$ does not differentiate to 0. Why is this not a contradiction?