Statistics 521, Problem Set 2
Wellner; 10/3/2012

Reading: Shorack, PfS, Chapter 2, pages 21 - 36.

Due: *Wednesday, October 10*, 2012.

Reminder: No class next Friday, October 12

1. PfS, Exercise 1.1.3, page 9 (and read the proof of the $\pi - \lambda$ theorem, Proposition 1.1.5, pages 9-10).

2. PfS, Exercise 1.2.1, page 15.

3. PfS, Exercise 1.2.3, page 16.

4. PfS, Exercise 1.2.4, page 16. (Nonmeasurable sets). Let Ω consist of the 16 values $1, \ldots, 16$. (Think of them arranged in four rows of four values.) Let

$$C_1 = \{1,2,3,4,5,6,7,8\}, \qquad C_2 = \{9,10,11,12,13,14,15,16\},$$
$$C_3 = \{1,2,5,6,9,10,13,14\}, \quad C_4 = \{3,4,7,8,11,12,15,16\}.$$

Let $\mathcal{C}$ denote the field generated by $\{C_1, C_2, C_3, C_4\}$, and let $\mathcal{A} = \sigma[\mathcal{C}]$.
(a) Show that $\mathcal{A} \equiv \sigma[\mathcal{C}] \neq 2^\Omega$. (Note that 2^Ω contains $2^{16} = 65,536$ sets.)
(b) Let $\mu(C_i) = 1/2$, $1 \le i \le 4$, with $\mu(C_1 C_3) = 1/4$. Show $\hat{\mathcal{A}}_\mu = \mathcal{A}$ with $2^4 = 16$ sets.
(c) Let $\mu(C_i) = 1/2$, $= 2,3,4$, with $\mu(C_2 C_4) = 0$. Show that $\hat{\mathcal{A}}_\mu$ has $2^{10} = 1024$ sets.
(d) Illustrate proposition 2.1 below in the context of this exercise.

5. PfS, Exercise A.1.5, page 428 (or PfS(2000), 9.1.5, page 182): verify (19), that F_{W_n} has derivative f_{W_n}.

6. **Optional bonus problem:** Let $\Omega = \mathbb{Z}$ =the integers, and let $\mathcal{A}$ be the collection of subsets A of $\mathbb{Z}$ so that A or A^c is finite. Let $\mu(A) = 0$ in the first case, and let $\mu(A) = 1$ in the second. Show that μ has no extension to $\sigma(\mathcal{A})$.