Statistics 521, Problem Set 9

Wellner; 11/21/2007

Reading:

- Shorack, PfS, Chapter 8, pages 165 - 171;
  Shorack, PfS, Chapter 10, pages 219 - 258.

Due: Wednesday, November 28, 2007

1. PfS, Exercise 5.3.1, page 94.
   Prove that:
   (a) A function $\underline{X} = (X_1, X_2, \ldots) : \Omega \to R^\infty$ is $\mathcal{B}_\infty - \mathcal{A}-$measurable if and only if each X_n is $\mathcal{B} - \mathcal{A}-$measurable.
   (b) If $\underline{X} = (X_1, X_2, \ldots)$ is $\mathcal{B}^\infty - \mathcal{A}-$ measurable and if $(i_1, i_2, \ldots)$ is an arbitrary sequence of integers, then $\underline{Y} \equiv (X_{i_1}, X_{i_2}, \ldots)$ is $\mathcal{B}^\infty - \mathcal{A}-$measurable.

2. PfS, Exercise 8.1.1, page 166:
   (a) Show that $P(AB) = P(A)P(B)$ if and only if $\{\emptyset, A, A^c, \Omega\}$ and $\{\emptyset, B, B^c, \Omega\}$ are independent $\sigma-$fields. (b) Show that $A_1, \ldots, A_n$ are independent if and only if (for each $k = 1, \ldots, n$,

$$P(A_{i_1} \ldots A_{i_k}) = \prod_{j=1}^{k} P(A_{i_j}) \qquad \text{whenever} \quad 1 \le i_1 < \cdots < i_k \le n\,,$$

   if and only if the $\sigma-$fields $\mathcal{A}_i \equiv \{\emptyset, A_i, A_i^c, \Omega\}$, with $1 \le i \le n$ are independent.

3. PfS, Exercise 8.1.3, page 168.

   If $\mathcal{C}_1, \ldots, \mathcal{C}_n$ are independent classes of events and if each $\mathcal{C}_i$ is a $\overline{\pi}$-system, then $\sigma[\mathcal{C}_1], \ldots, \sigma[\mathcal{C}_n]$ are independent $\sigma-$fields.

4. Give an example of two collections of sets $\mathcal{A}_1$ and $\mathcal{A}_2$ that are independent but the generated $\sigma-$fields are not independent.