Statistics 521, Problem Set 1

Wellner; 9/26/07

Reading: Shorack, PfS, Chapter 1, pages 1 - 17;
Due: Wednesday, October 3, 2007.

1. (a) Suppose that $\{\mathcal{A}_n\}$ is an increasing sequence of algebras, i.e. $\mathcal{A}_n \subset \mathcal{A}_{n+1}$ for all $n \geq 1$. Show that $\cup_{n=1}^{\infty}\mathcal{A}_n$ is an algebra.
(b) Suppose that the $\mathcal{A}_n$ of (a) are $\sigma-$algebras. Show by constructing a counter-example that $\cup_{n=1}^{\infty}\mathcal{A}_n$ need not be a $\sigma-$algebra.

2. Write out a proof of Proposition 1.1(b), PfS, page 3.

3. PfS, Exercise 1.1.1, PfS, page 4.

4. PfS, Exercise 1.1.2, PfS, page 8.

5. PfS(2000), Exercise 9.1.4, page 182. Suppose that $X_n \sim$ Binomial(n, p_n) where $np_n \to \lambda > 0$. Show that

$$P(X_n = k) \to \frac{\lambda^k}{k!}\exp(-\lambda) = P(Y = k)$$

where $Y \sim$ Poisson(λ); this implies that $X_n \to_d Y$.