MATH/STAT 394: Probability I
Wellner, 3/8/2000

Practice Final Exam

1. Suppose that $R \sim$ Exponential(μ), $S \sim$ Exponential(ν), and $T \sim$ Exponential(τ) are all independent. Suppose that $X \equiv R + T$, while $Y \equiv S + T$.
   (a) Compute $E(X)$, $Var(X)$, $E(Y)$, $Var(Y)$.
   (b) Compute $Cov(X, Y)$ and $\rho_{X,Y}$ the correlation between X and Y.
   (c) What is the joint density of R, S, T?

2. K, 6.4, # 7, page 382. Use the normal approximation to the binomial distribution to find the approximate probability that in 79 tosses of a fair coin there are fewer than 30 heads. Use a continuity correction.

3. K, 6.4, # 8, page 382, rephrased slightly: The event whose probability is found in Exercise 7, "fewer than 30 heads in 79 tosses" can be restated as "80 or more trials are needed to produce the 30th head"; i.e. in the notation of our section on the Bernoulli process, $[T_{79} < 30] = [W_{30} > 79]$. You can approximate the probability of the event on the right side by using the Central Limit Theorem. Do this and compare with the result of the previous problem.

4. Suppose that the marginal distribution of X is Exponential(2), and the conditional distribution of Y given $X = x$ is Exponential(x).
   (a) Determine the joint density of X and Y.
   (b) Determine the marginal density of Y.
   (c) Determine the conditional density of X given $Y = y$.

5. Suppose that $X_1, X_2, \ldots, X_{12}$ are independent coins flips (Bernoulli(1/2) random variables). Let $T_1 = X_1 + X_2 + X_3 + X_4$, $T_2 = X_5 + X_6 + X_7 + X_8$, and $T_3 = X_9 + X_{10} + X_{11} + X_{12}$. Evaluate:
   (a) $E(T_1 + T_3)$. (b) $Var(T_1 + T_3)$.
   (c) $Cov(T_1 + T_2, T_2 + T_3)$. (d) What is the distribution of $T_1 + T_3$?

6. Customers arrive at the check-out area of Larry's Market at a rate of 2 per minute. Break the upcoming hours into six disjoint ten minute intervals. "Customer overload" is said to occur if such a ten minute interval sees the arrival of at least 27 customers.
   A. What is the probability of:
   (a) Customer overload in the first interval?
   (b) The first hour?
   (c) The second customer overload occuring in the eleventh interval?
   B. Evaluate the mean and variance of the number of turns until the tenth overload.

7. Roll a pair of dice (one read and one white) with outcomes X_1 and X_2. Let $T = X_1 + X_2$ and $D = |X_1 - X_2|$. Evaluate:
   (a) $P(T \geq 6 | X_1 \times X_2 \leq 8)$.
   (b) $P(X_1 + X_2 \leq 8 | T \geq 6)$.
   (c) $P(T \geq 8 | D \leq 1)$.

8. A and B take turns rolling two dice with A going first. A wins if he rolls "5" or "9", while B wins if he rolls "6" or "8". Determine $P(A \text{ wins})$ in this sequential contest.

9. Consider drawing (without replacement) from an urn consisting of 52 balls, four of which are labeled with the number 5, twelve of which are labeled with the number 2, and thirty-six of which are labeled with the number 1. Determine the mean and standard deviation of:
   (a) One randomly chosen ball.
   (b) A sample of 5 randomly chosen (without replacement) balls.

10. Evaluate (or approximate) as accurately as you can, the probability of at least 6 honor cards in at least 20 of 100 hands of bridge.