SISMID 2020 Spatial Statistics Waller Point Process 1

Lance A. Waller

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• What we have

- Event locations for a strand of 327 myrtle trees in a rectangular plot 170.5×213.0 meters.
- 221 healthy trees.
- 106 diseased trees.
- Research question: Is the spatial pattern of diseased trees the same as the spatial pattern of health trees?

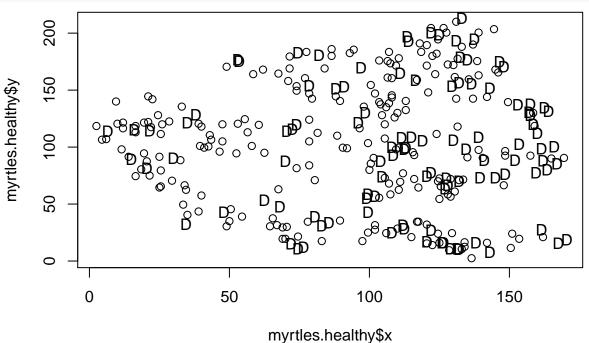
• Reading in the data, basic R commands

[1] "x" "y"

[1] 221

• Plotting the data

- Read in data on diseased trees.
- Plot patterns
- Take care to make *square* plots, covering the *same* area. ***

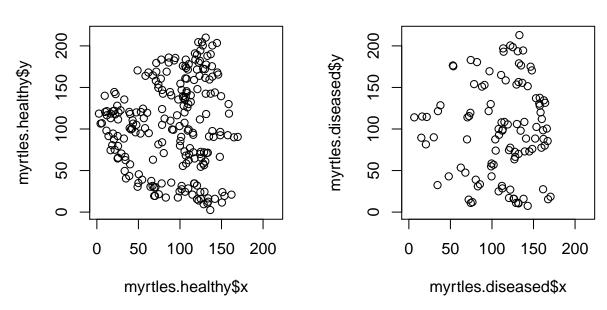


```
# Notice that the plot is sort of square, but
\# that the range of values for x is different
# from that for y. Let's set the limits
# so that they are the same. First, we find
# min and max of the x and y coordinates for
# ALL myrtle locations (healthy and diseased).
min(myrtles.all$x)
## [1] 2.5
max(myrtles.all$x)
## [1] 170.5
min(myrtles.all$y)
## [1] 2.5
max(myrtles.all$y)
## [1] 213
# We can also use the "range" command
# to do this.
range(myrtles.all$x)
## [1]
       2.5 170.5
range(myrtles.all$y)
## [1]
       2.5 213.0
# Looks like if we set the plot boundaries
# for (0,215) for x and y, we'll catch all
# of the points. We use the "xlim" and
# "ylim" parameters in the plot command.
# NOTE: we can continue a command onto the next
# line if we end with a comma and don't include
# a closing paranthesis until we are ready.
# ALSO NOTE: "c(0,215)" concatenates the values
# 0 and 215 into a vector.
plot(myrtles.healthy$x,myrtles.healthy$y,xlim=c(0,215),
    ylim=c(0,215))
# Finally, to make sure R draws the plotting area
# as a square, we introduce the "par" command.
# "par" sets plotting parameters and is a very,
# very, very, very, very, very important
```

```
# command with lots of uses. You have to set
# "par" before plotting, but the settings stay until
# the next "par" command resets them.
# "pty" = "plot type" and "pty=s" means "set plot type
# to square".
par(pty="s")
plot(myrtles.healthy$x,myrtles.healthy$y,xlim=c(0,215),
     ylim=c(0,215))
# We can also use "par" to put put multiple plots
# in the same window.
# "mfrow" means "multiple figures by row".
# "mfrow=c(1,2)" means "multiple\ figures, one row
# containing two figures". Let's try it.
par(pty="s",mfrow=c(1,2))
plot(myrtles.healthy$x,myrtles.healthy$y,xlim=c(0,215),
     ylim=c(0,215))
title("Healthy")
plot(myrtles.diseased$x,myrtles.diseased$y,xlim=c(0,215),
     ylim=c(0,215))
title("Diseased")
```



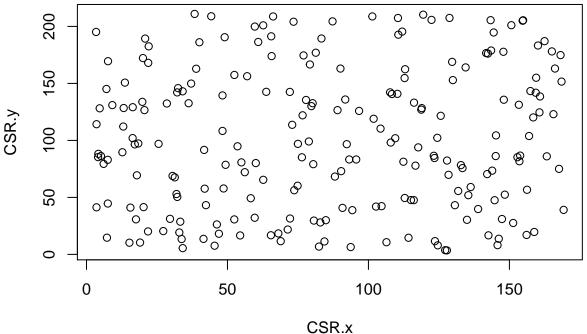
Diseased



- Let's test for CSR
- Consider test statistic by Pielou (1959)
- Test statistic $P = \pi \lambda \sum X_i^2/n$

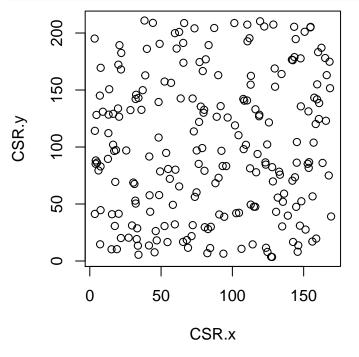
- X_i = distance from event i to its nearest neighbor
- Pielou (1959) suggests $P \stackrel{a}{\sim} N(1, 1/n)$.\ [link] (http://methodsblog.com/2017/03/10/ec-pielou/)
- Let's try a Monte Carlo test.

```
# To generate realizations from CSR in the range
# of values of the data we use "runif"
# command that generates uniformly distributed
# random numbers.
# NOTE: We don't want to generate on the interval
# (0,215) since we want to limit the range of values
# to the range of the data. We extended the region to
# get a square plot, but we want to limit simulations
# to the area with data.
# Let's set the number of events to simulate to
# match the observed number of events.
num.events = length(myrtles.healthy$x)
CSR.x <- runif(num.events,min(myrtles.all\$x),max(myrtles.all\$x))
CSR.y <- runif(num.events,min(myrtles.all$y),max(myrtles.all$y))
plot(CSR.x,CSR.y)
```



• Need to reset plot to be in a square

```
par(mfrow=c(1,1),pty="s")
plot(CSR.x,CSR.y)
```



For a Monte Carlo test, we will want to make a loop of CSR simulations and calculate the test statistic for each simulated realization.

Let's calculate a clustering statistic due to Pielou (1959). This statistic requires us to find the distance from each event to its nearest neighbor. (This also gives us a chance to try out some other R functions.)

First, for each event, calculate the distance to all other events.

We access individual x or y values by brackets with the index, i.e., x[1] is the first element of x.

We can find the distance between (x[1],y[1]) and all other observations by...

Now we want to find the minimum element of "dist1" that is NOT 0. We can use a nifty feature of R namely we can put logical expressions inside brackets and get only the elements where that expression is true. For instance,

```
# Commented out so it doesn't list output
# dist1[dist1!=0]
```

gives the elements of dist1 that are not equal to zero. So

```
min(dist1[dist1!=0])
## [1] 4.472136
gives the nearest neighbor distance! Now we just need it for all values. Let's use a loop to get this.
###################
# Set a vector of zeros with length equal to the number of locations in myrtles.all (length(myrtles.all
mindist.all <- 0*(1:length(myrtles.all$x))</pre>
for (i in 1:length(myrtles.all$x)) {
 dist = sqrt( (myrtles.all\$x[i] - myrtles.all\$x)^2 + (myrtles.all\$y[i] - myrtles.all\$y)^2 )
  mindist.all[i] = min(dist[dist!=0])
}
Now we need to calculate Pielou's statistic.
n.all = length(myrtles.all$x)
area.all = (max(myrtles.all\$x) - min(myrtles.all\$x)) * (max(myrtles.all\$y)-min(myrtles.all\$y))
lambda.all = n.all/area.all
pielou.all = pi*lambda.all*sum(mindist.all^2)/n.all
# Print out the value using 'paste' to put text before it. (sep="" to have no
# separater between the text and the value)
print(paste("Pielou's statistic value = ",pielou.all,sep=""))
## [1] "Pielou's statistic value = 0.65680109594231"
Let's round the statistic to make the print output a little cleaner.
print(paste("Pielou's statistic value = ",round(pielou.all,4),sep=""))
## [1] "Pielou's statistic value = 0.6568"
Now to set up the Monte Carlo test
To get a Monte Carlo p-value we need todo these same calculations to data generated under CSR. First define
the number of simulations.
num.sim = 99
then define a vector to hold the values of Pielou's statistic for each simulated data set.
pielou.all.sim = 0*(1:num.sim)
Now set up the simulation loop
for (sim in 1:num.sim) {
  # define CSR data
```

CSR.x = runif(num.events,min(myrtles.all\$x),max(myrtles.all\$x))

```
CSR.y = runif(num.events,min(myrtles.all$y),max(myrtles.all$y))

#define vector of min NN distances
mindist.sim = 0*(1:length(myrtles.all$x))

# find min distances (a loop within the simulation loop)

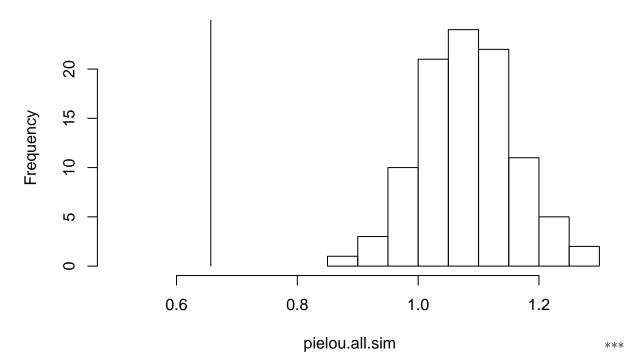
for (i in 1:length(CSR.x)) {
    dist = sqrt( (CSR.x[i] - CSR.x)^2 + (CSR.y[i] - CSR.y)^2 )
    mindist.sim[i] = min(dist[dist!=0])
}

# calculate pielou.all.sim[sim] (Pielou's statistic for the "sim-th" CSR data set).
pielou.all.sim[sim] = pi*lambda.all*sum(mindist.sim^2)/n.all
}
```

Make a histogram of the CSR values

```
par(pty="m") # makes plot type "maximum" (rectangular in window).
hist(pielou.all.sim,xlim=c(0.5,max(pielou.all.sim)))
# add a vertical line showing Pielou's statistic from the observed data.
# ("segments(x1,y1,x2,y2)" draws a line segments between (x1,y1) and (x2,y2).
segments(pielou.all,0,pielou.all,100)
```

Histogram of pielou.all.sim



Now we can calculate Monte Carlo p-value (the number of statistics from simulated data that exceed the statistic from the observed data divided by the number of simulations +1.

```
p.val = length(pielou.all.sim[pielou.all.sim>pielou.all])/(num.sim+1)
print(paste("Peilou's statistic: ",round(pielou.all,4)," p-val = ",round(p.val,4),sep=""))
## [1] "Peilou's statistic: 0.6568 p-val = 0.99"
Now to do this for the healthy and diseased subsets
mindist.healthy = 0*(1:length(myrtles.healthy$x))
for (i in 1:length(myrtles.healthy$x)) {
     dist = sqrt((myrtles.healthy x[i] - myrtles.healthy x)^2 + (myrtles.healthy y[i] - myrtles.healthy y[i] - myrtle
     mindist.healthy[i] = min(dist[dist!=0])
}
n.healthy = length(myrtles.healthy$x)
# NOTE: We still use area.all to cover the entire study area.
lambda.healthy = n.healthy/area.all
pielou.healthy = pi*lambda.healthy*sum(mindist.healthy^2)/n.healthy
print(paste("Peilou's statistic, healthy myrtles:",pielou.healthy))
## [1] "Peilou's statistic, healthy myrtles: 0.657911544652011"
Now for the diseased trees
mindist.diseased = 0*(1:length(myrtles.diseased$x))
for (i in 1:length(myrtles.diseased$x)) {
    dist = sqrt( (myrtles.diseased$x[i] - myrtles.diseased$x^2 + (myrtles.diseased$y[i] - myrtles.diseased$y
    mindist.diseased[i] = min(dist[dist!=0])
}
n.diseased = length(myrtles.diseased$x)
# NOTE: We still use area.all to cover the entire study area.
lambda.diseased = n.diseased/area.all
pielou.diseased = pi*lambda.diseased*sum(mindist.diseased^2)/n.diseased
print(paste("Peilou's statistic, diseased myrtles:",pielou.diseased))
## [1] "Peilou's statistic, diseased myrtles: 0.507452851359244"
What we have: Separate tests for healthy trees and for diseased trees.
```

What we don't have: A comparison between healthy and diseased trees.