

2021 SISCER APC Course, R Notes Set 2

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Overview of R notes

In these notes we will continue to analyze data on male lung cancer mortality in Denmark, for the period 1943–1996 in ages 40–89. Data are in the `Epi` package. Age, period and cohort effects will all be modeled in 5-year intervals.

We will illustrate:

- ▶ Factor and spline models in the `Epi` package (Carstensen)
- ▶ The convenient parameterization in the `apc` package (Nielsen)

Danish male lung cancer incidence data

Recall, from the data description:

A data frame with 220 observations on the following 9 variables.

- ▶ A5: Left end point of the age interval, a numeric vector.
- ▶ P5: Left end point of the period interval, a numeric vector.
- ▶ C5: Left end point of the birth cohort interval, a numeric vector.
- ▶ up: Indicator of upper triangles of each age by period rectangle in the Lexis diagram. ($up = (P5 - A5 - C5) / 5$).
- ▶ Ax: The mean age of diagnosis (at risk) in the triangle.
- ▶ Px: The mean date of diagnosis (at risk) in the triangle.
- ▶ Cx: The mean date of birth in the triangle, a numeric vector.
- ▶ D: Number of diagnosed cases of male lung cancer.
- ▶ Y: Risk time in the male population, person-years.

Massaging the data into a convenient form

```
library(Epi)
data(lungDK)
attach(lungDK)
dftempEpi = data.frame(D = lungDK$D, Y = lungDK$Y,
  A = 37.5 + 5 * ((lungDK$A5 - min(lungDK$A5))/5 +
    1), P = 1945.5 + 5 * (lungDK$P5 - min(lungDK$P5))/5 +
    1)
```

Massaging the data into a convenient form

Sum over the upper and lower triangles in the Lexis diagram to get data in age-period squares, see Carstensen (2007).

```
names(dftempEpi)
## [1] "D" "Y" "A" "P"
head(dftempEpi, 2)
##      D      Y      A      P
## 1 52 336233.8 42.5 1946.5
## 2 28 357812.7 42.5 1946.5
# Sum over upper and lower triangles
dfEpi = aggregate(dftempEpi[, c("D", "Y")], by = list(A = dftempEpi$A, P = dftempEpi$P), sum)
head(dfEpi, 2)
##      A      P      D      Y
## 1 42.5 1946.5 80 694046.5
## 2 47.5 1946.5 135 622256.7
```

Fitting a sequence of factor models in the Epi package

Recall there are $P = 11$ periods and $A = 10$ age groups and so $A + P - 1 = 20$ cohorts.

```
fit1 <- apc.fit(dfEpi, model = "factor", parm = "ACP",
  scale = 10^5)
## [1] "ML of APC-model Poisson with log(Y) offset : ( ACP ):\n"
##           Model Mod. df.  Mod. dev. Test df. Test dev.      Pr(>Chi)
## 1           Age      100 15103.0013      NA      NA      NA
## 2      Age-drift      99  6417.3811       1 8685.6202 0.000000e+00
## 3      Age-Cohort      81   829.6293      18 5587.7518 0.000000e+00
## 4 Age-Period-Cohort      72   208.5476       9  621.0817 6.244637e-128
## 5      Age-Period      90  2723.4660      18 2514.9184 0.000000e+00
## 6      Age-drift      99  6417.3811       9 3693.9151 0.000000e+00
## Test dev/df      H0
## 1           NA
## 2 8685.62024 zero drift
## 3 310.43065 Coh eff/dr.
## 4  69.00907 Per eff/Coh
## 5 139.71769 Coh eff/Per
## 6  410.43501 Per eff/dr.
## No reference cohort given; reference cohort for age-effects is chosen as
## the median date of birth for persons with event: 1914 .
```

Factor models

The function `apc.fit` fits a sequence of models.

Notice the `model="factor"` to give a different level for each of age, period and cohort, later we will present alternative smoother (spline) models.

For comparison we fit models using the `glm` function.

For example, for the Age+Period model we have factors for age and period, $E[Y_{ap}] = N_{ap} \exp(\delta + \alpha_a + \beta_p)$.

The fits are the same (e.g., compare deviances and degrees of freedom), but the parameterizations are different.

The default corner-point parameterization in the GLM version takes the first groups as reference, i.e., $\alpha_1 = \beta_1 = 0$.

Factor models $A = 10, P = 11, C = A + P - 1 = 20$

Degrees of freedom and parameters:

- ▶ There are $11 \times 10 = 110$ total degrees of freedom (DF)
- ▶ Age only model has 10 parameters and so 100 Residual DF
- ▶ Age-drift model has $10 + 1 = 11$ parameters and so 99 Residual DF
- ▶ Age-Cohort model has $10 + 19 = 29$ parameters and so 81 Residual DF
- ▶ Age-Period-Cohort model has $10 + 19 + 11 - 2 = 38$ parameters and so 72 Residual DF
- ▶ Age-Period model has $10 + 11 - 1 = 20$ parameters and so 90 Residual DF

Factor models

The cohort drift model provides an identical fit to the period drift model because of the linear dependence relationship between period and cohort: $c = A - a + p$.

In terms of significance, the counts are very large and so very small deviations from the null are detectable.

Everything is significant!

Factor APC models in Epi

```
names(fit1)
## [1] "Type" "Model" "Age" "Per" "Coh" "Drift" "Ref" "Anova"
dim(fit1$Coh)
## [1] 20 4
dim(fit1$Per)
## [1] 11 4
dim(fit1$Age)
## [1] 10 4
fit1$Ref
## Per Coh
## NA 1914
```

Factor APC models in Epi

```
head(fit1$Coh, n = 3)
```

```
##      Coh      C-RR      2.5%      97.5%  
## 10 1859 0.07960723 0.03785991 0.1673884  
## 9  1864 0.07939452 0.05546108 0.1136561  
## 8  1869 0.13101490 0.11055245 0.1552648
```

```
head(fit1$Per, n = 3)
```

```
##      Per      P-RR      2.5%      97.5%  
## 1  1946.5 0.8527416 0.8144673 0.8928145  
## 11 1951.5 0.9344356 0.8994733 0.9707568  
## 21 1956.5 0.9395033 0.9085332 0.9715291
```

```
head(fit1$Age, n = 3)
```

```
##      Age      Rate      2.5%      97.5%  
## 1  42.5  9.013028  8.44474  9.619559  
## 2  47.5 24.453671 23.47509 25.473051  
## 3  52.5 59.161763 57.52394 60.846220
```

Age factor model, using the glm function

```
glmMA <- glm(D ~ as.factor(A) + offset(log(Y)), data = dfEpi,  
             family = "poisson")  
glmMA  
##  
## Call:  glm(formula = D ~ as.factor(A) + offset(log(Y)), family = "poisson",  
##       data = dfEpi)  
##  
## Coefficients:  
##      (Intercept)  as.factor(A)47.5  as.factor(A)52.5  as.factor(A)57.5  
##      -9.0172      0.9504      1.7841      2.4288  
## as.factor(A)62.5  as.factor(A)67.5  as.factor(A)72.5  as.factor(A)77.5  
##      2.8938      3.1962      3.3750      3.3788  
## as.factor(A)82.5  as.factor(A)87.5  
##      3.2636      3.0228  
##  
## Degrees of Freedom: 109 Total (i.e. Null); 100 Residual  
## Null Deviance:      71780  
## Residual Deviance: 15100      AIC: 15980
```

Age factor model, using the glm function

Notice that the residual DF and deviance are identical to the previous fit.

```
glmMA$deviance
```

```
## [1] 15103
```

```
glmMA$df.residual
```

```
## [1] 100
```

```
fit1$Anova[1, ]
```

```
##      Model Mod. df. Mod. dev. Test df. Test dev. Pr(>Chi) 1
```

```
## 1      Age      100      15103      NA      NA      NA
```

Age-Period factor model, using the glm function

Similarly for the age-period model

```
glmMAP <- glm(D ~ as.factor(A) + as.factor(P) + offset(log(Y)),  
  data = dfEpi, family = "poisson")  
glmMAP$df.residual  
## [1] 90  
glmMAP$deviance  
## [1] 2723.466  
fit1$Anova[5, ]  
##           Model Mod. df. Mod. dev. Test df. Test dev. Pr(>Chi) T  
## 5 Age-Period           90  2723.466           18  2514.918           0  
##           H0  
## 5 Coh eff/Per
```

Period drift model, using the glm function

```
glmMdrift <- glm(D ~ as.factor(A) + P + offset(log(Y)),
  data = dfEpi, family = "poisson")
glmMdrift
##
## Call:  glm(formula = D ~ as.factor(A) + P + offset(log(Y)), family = "poisson",
##      data = dfEpi)
##
## Coefficients:
##      (Intercept)  as.factor(A)47.5  as.factor(A)52.5  as.factor(A)57.5
##      -55.05841      0.94972      1.79355      2.44053
## as.factor(A)62.5  as.factor(A)67.5  as.factor(A)72.5  as.factor(A)77.5
##      2.89467      3.18094      3.34283      3.33145
## as.factor(A)82.5  as.factor(A)87.5      P
##      3.19513      2.93045      0.02331
##
## Degrees of Freedom: 109 Total (i.e. Null); 99 Residual
## Null Deviance:      71780
## Residual Deviance: 6417  AIC: 7296
```

Period drift model, using the glm function

```
glmMdrift$df.residual
## [1] 99
glmMdrift$deviance
## [1] 6417.381
exp(coefficients(glmMdrift)["P"])
##      P
## 1.02358
fit1[["Drift"]]
##               exp(Est.)      2.5%      97.5%
## APC (Y-weights)  1.021348 1.020444 1.022253
## A-d              1.023580 1.023065 1.024096
```

Under the age-drift model there is an estimated 1.0235804% increase in mortality per cohort.

Age-Cohort model

We create the cohort factors and then fit a GLM to show the equivalence in terms of fit.

```
Cterm <- dfEpi$P - dfEpi$A
sort(unique(Cterm))
## [1] 1859 1864 1869 1874 1879 1884 1889 1894 1899 1904 1909 1914
## [16] 1934 1939 1944 1949 1954
length(unique(Cterm))
## [1] 20
```

Recall $A = 10$, $P = 11$ and so $C = A + P - 1 = 20$.

Age-Cohort factor model

```
glmMAC <- glm(D ~ as.factor(A) + as.factor(Cterm) +  
  offset(log(Y)), data = dfEpi, family = "poisson")  
glmMAC$df.residual  
## [1] 81  
glmMAC$deviance  
## [1] 829.6293
```

Age-Period factor model

Now fit two Age+Period factor models using default parameterization, and fixing a particular level for the period effects.

```
APv1 <- glm(D ~ factor(A) + factor(P) + offset(log(Y)),  
            family = poisson, data = dfEpi)  
APv1$df.residual  
## [1] 90  
APv1$deviance  
## [1] 2723.466  
APv2 <- glm(D ~ factor(A) + relevel(factor(P), "1971.5") +  
            offset(log(Y)), family = poisson, data = dfEpi)  
APv2$df.residual  
## [1] 90  
APv2$deviance  
## [1] 2723.466
```

Age Period factor model

```
summary(APv1)
```

```
##
## Call:
## glm(formula = D ~ factor(A) + factor(P) + offset(log(Y)), family = poisson,
##      data = dfEpi)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -10.400   -3.728   -0.984    3.685   11.203
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)   -10.34235    0.04192  -246.71  <2e-16 ***
## factor(A)47.5    0.95258    0.03673   25.93  <2e-16 ***
## factor(A)52.5    1.78237    0.03383   52.69  <2e-16 ***
## factor(A)57.5    2.41412    0.03265   73.94  <2e-16 ***
## factor(A)62.5    2.86259    0.03216   89.01  <2e-16 ***
## factor(A)67.5    3.15159    0.03201   98.47  <2e-16 ***
## factor(A)72.5    3.31784    0.03209  103.40  <2e-16 ***
## factor(A)77.5    3.30980    0.03261  101.50  <2e-16 ***
## factor(A)82.5    3.17640    0.03423   92.81  <2e-16 ***
## factor(A)87.5    2.90983    0.04024   72.32  <2e-16 ***
## factor(P)1951.5  0.39206    0.03629   10.80  <2e-16 ***
## factor(P)1956.5  0.67592    0.03404   19.86  <2e-16 ***
## factor(P)1961.5  1.01434    0.03226   31.44  <2e-16 ***
## factor(P)1966.5  1.26666    0.03130   40.47  <2e-16 ***
## factor(P)1971.5  1.48717    0.03067   48.49  <2e-16 ***
## factor(P)1976.5  1.59239    0.03039   52.40  <2e-16 ***
## factor(P)1981.5  1.67994    0.03020   55.62  <2e-16 ***
## factor(P)1986.5  1.69902    0.03015   56.35  <2e-16 ***
## factor(P)1991.5  1.59958    0.03028   52.83  <2e-16 ***
## factor(P)1996.5  1.52558    0.03078   49.57  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
```

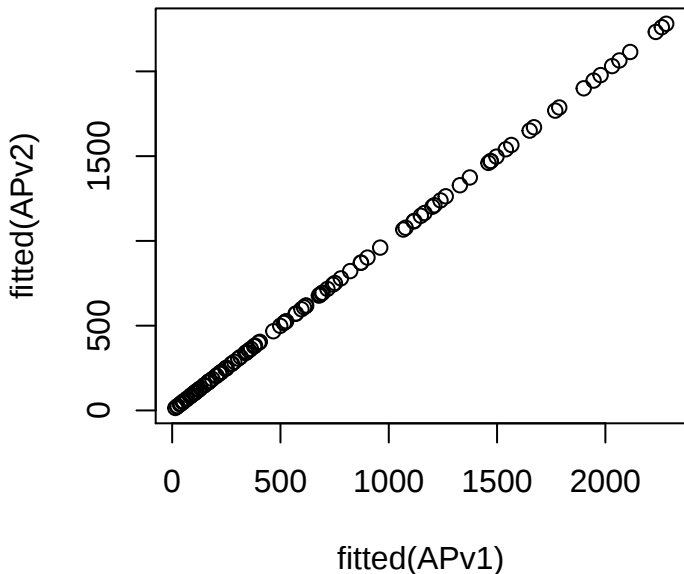
Age Period factor model

```
summary(APv2)
```

```
##
## Call:
## glm(formula = D ~ factor(A) + relevel(factor(P), "1971.5") +
##      offset(log(Y)), family = poisson, data = dfEpi)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -10.400   -3.728   -0.984    3.685   11.203
##
## Coefficients:
##
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)      -8.85517    0.03267 -271.034 < 2e-16 ***
## factor(A)47.5       0.95258    0.03673  25.932 < 2e-16 ***
## factor(A)52.5       1.78237    0.03383  52.692 < 2e-16 ***
## factor(A)57.5       2.41412    0.03265  73.938 < 2e-16 ***
## factor(A)62.5       2.86259    0.03216  89.007 < 2e-16 ***
## factor(A)67.5       3.15159    0.03201  98.468 < 2e-16 ***
## factor(A)72.5       3.31784    0.03209 103.401 < 2e-16 ***
## factor(A)77.5       3.30980    0.03261 101.497 < 2e-16 ***
## factor(A)82.5       3.17640    0.03423  92.805 < 2e-16 ***
## factor(A)87.5       2.90983    0.04024  72.316 < 2e-16 ***
## relevel(factor(P), "1971.5")1946.5 -1.48717    0.03067 -48.493 < 2e-16 ***
## relevel(factor(P), "1971.5")1951.5 -1.09512    0.02481 -44.134 < 2e-16 ***
## relevel(factor(P), "1971.5")1956.5 -0.81125    0.02137 -37.958 < 2e-16 ***
## relevel(factor(P), "1971.5")1961.5 -0.47284    0.01842 -25.674 < 2e-16 ***
## relevel(factor(P), "1971.5")1966.5 -0.22051    0.01667 -13.227 < 2e-16 ***
## relevel(factor(P), "1971.5")1976.5  0.10522    0.01488  7.071 1.54e-12 ***
## relevel(factor(P), "1971.5")1981.5  0.19276    0.01449 13.300 < 2e-16 ***
## relevel(factor(P), "1971.5")1986.5  0.21184    0.01439 14.724 < 2e-16 ***
## relevel(factor(P), "1971.5")1991.5  0.11241    0.01465  7.670 1.71e-14 ***
## relevel(factor(P), "1971.5")1996.5  0.03840    0.01566  2.453 0.0142 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
```

Age-Period Model: Fits are the same

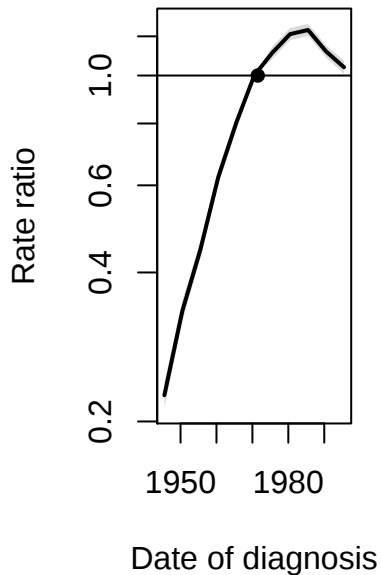
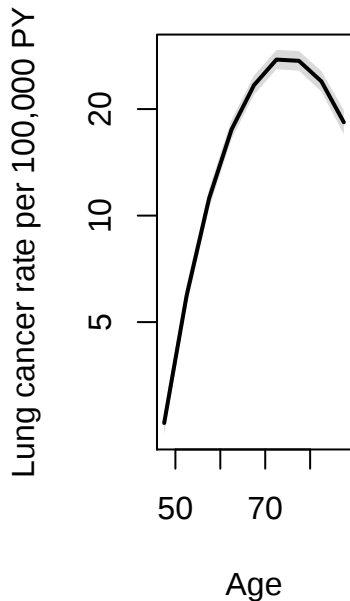
```
plot(fitted(APv2) ~ fitted(APv1))
```



Age-Period factor model: Chosen Parameterization

```
par(mfrow = c(1, 2))
matshade(seq(47.5, 87.5, 5), ci.exp(APv2, subset = "A"),
  plot = TRUE, log = "y", lwd = 2, xlab = "Age",
  ylab = "Lung cancer rate per 100,000 PY")
matshade(seq(1945.5, 1996.5, 5), rbind(ci.exp(APv2,
  subset = "P")[1:5, ], 1, ci.exp(APv2, subset = "P")[6:10,
  ]), plot = TRUE, log = "y", lwd = 2, xlab = "Date of diagnosis",
  ylab = "Rate ratio")
abline(h = 1)
points(1971.5, 1, pch = 16)
```

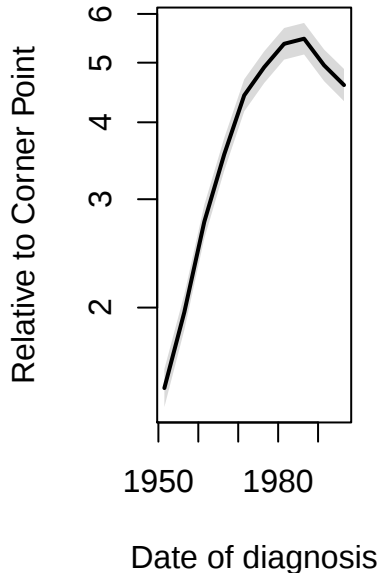
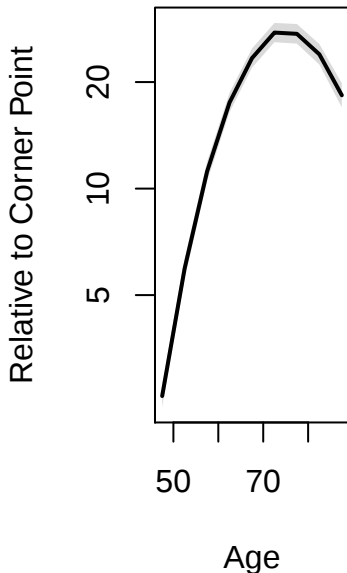
Age-Period factor model: Chosen Parameterization



Age-Period factor model: Default Parameterization

```
par(mfrow = c(1, 2))
matshade(seq(47.5, 87.5, 5), ci.exp(APv1, subset = "A"),
  plot = TRUE, log = "y", lwd = 2, xlab = "Age",
  ylab = "Relative to Corner Point")
matshade(seq(1951.5, 1996.5, 5), ci.exp(APv1, subset = "P"),
  plot = TRUE, log = "y", lwd = 2, xlab = "Date of diagnosis",
  ylab = "Relative to Corner Point")
abline(h = 1)
points(1971.5, 1, pch = 16)
```

Age-Period factor model: Default Parameterization



Age cohort drift model

```
glmMdrift2 <- glm(D ~ as.factor(A) + Cterm + offset(log(Y)),
  data = dfEpi, family = "poisson")
glmMdrift2$df.residual
## [1] 99
glmMdrift2$deviance
## [1] 6417.381
coef(glmMdrift2)
##      (Intercept) as.factor(A)47.5 as.factor(A)52.5 as.factor(A)57.5
##      -54.0678788      1.0662500      2.0266201      2.7901274
## as.factor(A)62.5 as.factor(A)67.5 as.factor(A)72.5 as.factor(A)77.5
##      3.3608044      3.7636050      4.0420347      4.1471891
## as.factor(A)82.5 as.factor(A)87.5      Cterm
##      4.1274006      3.9792565      0.0233067
```

Fitted curves under a particular parameterization

```
apc.plot(fit1)
```

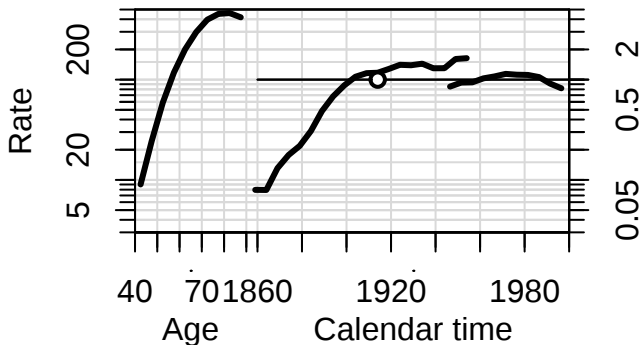


Figure 1: Age-period-cohort estimates under ACP factor model, “fit1”.

```
## cp.offset    RR.fac  
##      1765      100
```

Full APC Model with Poisson Likelihood

```
glmMACP <- glm(D ~ as.factor(A) + as.factor(Cterm) +  
  as.factor(P) + offset(log(Y)), data = dfEpi, family = "poisson")  
glmMACP$df.residual  
## [1] 72  
glmMACP$deviance  
## [1] 208.5476  
tail(coef(glmMACP))  
## as.factor(P)1971.5 as.factor(P)1976.5 as.factor(P)1981.5 as.factor(P)1986.5  
##          0.3110585          0.2959105          0.2944408          0.2490253  
## as.factor(P)1991.5 as.factor(P)1996.5  
##          0.1031232          NA
```

Note that 1 parameter estimate is NA because the model is over parameterized.

Full APC Model with quasi-Poisson Likelihood

Variance is $\kappa \times \mu$.

```
glmMACPexcess <- glm(D ~ as.factor(A) + as.factor(Cterm) +  
  as.factor(P) + offset(log(Y)), data = dfEpi, family = "quasipoisson")  
SSpears <- sum(residuals(glmMACPexcess, type = "pearson")^2)  
excess <- SSpears/glmMACPexcess$df.residual  
excess  
## [1] 2.943842
```

Quite a lot of overdispersion here, $\hat{\kappa} = 2.9438419$.

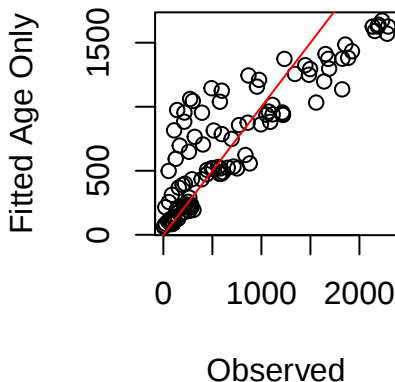
Full ABC model with quasi-Poisson likelihood

```
summary(glmMACPexcess)
```

```
##
## Call:
## glm(formula = D ~ as.factor(A) + as.factor(Cterm) + as.factor(P) +
##       offset(log(Y)), family = "quasipoisson", data = dfEpi)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -4.0478  -0.8303   0.0357   0.8045   3.1655
##
## Coefficients: (1 not defined because of singularities)
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -11.968761    0.654109 -18.298 < 2e-16 ***
## as.factor(A)47.5     0.994171    0.065405  15.200 < 2e-16 ***
## as.factor(A)52.5     1.873728    0.062463  29.997 < 2e-16 ***
## as.factor(A)57.5     2.559631    0.062015  41.275 < 2e-16 ***
## as.factor(A)62.5     3.087159    0.062756  49.193 < 2e-16 ***
## as.factor(A)67.5     3.479003    0.064363  54.053 < 2e-16 ***
## as.factor(A)72.5     3.760496    0.066622  56.446 < 2e-16 ***
## as.factor(A)77.5     3.888688    0.069775  55.732 < 2e-16 ***
## as.factor(A)82.5     3.903883    0.074581  52.344 < 2e-16 ***
## as.factor(A)87.5     3.801889    0.085495  44.469 < 2e-16 ***
## as.factor(Cterm)1864 -0.006614    0.721242  -0.009 0.992709
## as.factor(Cterm)1869  0.490330    0.666741   0.735 0.464477
## as.factor(Cterm)1874  0.789467    0.656304   1.203 0.232956
## as.factor(Cterm)1879  0.994787    0.653326   1.523 0.132226
## as.factor(Cterm)1884  1.330029    0.651900   2.040 0.044996 *
## as.factor(Cterm)1889  1.786722    0.651097   2.744 0.007653 **
## as.factor(Cterm)1894  2.119172    0.650737   3.257 0.001721 **
## as.factor(Cterm)1899  2.367836    0.650552   3.640 0.000510 ***
## as.factor(Cterm)1904  2.560970    0.650471   3.937 0.000188 ***
## as.factor(Cterm)1909  2.640060    0.650475   4.059 0.000124 ***
## as.factor(Cterm)1914  2.645518    0.650605   4.066 0.000121 ***
## as.factor(Cterm)1919  2.728574    0.650700   4.193 7.72e-05 ***
## as.factor(Cterm)1924  2.819711    0.650847   4.332 4.70e-05 ***
## as.factor(Cterm)1929  2.806058    0.651161   4.309 5.10e-05 ***
## as.factor(Cterm)1934  2.887553    0.651353   4.435 1.22e-05 ***
## as.factor(Cterm)1939  2.970058    0.651545   4.559 2.95e-05 ***
## as.factor(Cterm)1944  3.052563    0.651737   4.683 6.40e-05 ***
## as.factor(Cterm)1949  3.135068    0.651929   4.807 1.59e-05 ***
## as.factor(Cterm)1954  3.217573    0.652121   4.931 3.90e-05 ***
## as.factor(Cterm)1959  3.300078    0.652313   5.055 9.30e-05 ***
## as.factor(Cterm)1964  3.382583    0.652505   5.179 2.17e-05 ***
## as.factor(Cterm)1969  3.465088    0.652697   5.303 5.10e-05 ***
## as.factor(Cterm)1974  3.547593    0.652889   5.427 1.22e-05 ***
## as.factor(Cterm)1979  3.630098    0.653081   5.551 2.95e-05 ***
## as.factor(Cterm)1984  3.712603    0.653273   5.675 6.40e-05 ***
## as.factor(Cterm)1989  3.795108    0.653465   5.799 1.59e-05 ***
## as.factor(Cterm)1994  3.877613    0.653657   5.923 3.90e-05 ***
## as.factor(Cterm)1999  3.960118    0.653849   6.047 9.30e-05 ***
## as.factor(Cterm)2004  4.042623    0.654041   6.171 2.17e-05 ***
## as.factor(Cterm)2009  4.125128    0.654233   6.295 5.10e-05 ***
## as.factor(Cterm)2014  4.207633    0.654425   6.419 1.22e-05 ***
## as.factor(Cterm)2019  4.290138    0.654617   6.543 2.95e-05 ***
## as.factor(Cterm)2024  4.372643    0.654809   6.667 6.40e-05 ***
## as.factor(Cterm)2029  4.455148    0.654999   6.791 1.59e-05 ***
## as.factor(Cterm)2034  4.537653    0.655191   6.915 3.90e-05 ***
## as.factor(Cterm)2039  4.620158    0.655383   7.039 9.30e-05 ***
## as.factor(Cterm)2044  4.702663    0.655575   7.163 2.17e-05 ***
## as.factor(Cterm)2049  4.785168    0.655767   7.287 5.10e-05 ***
## as.factor(Cterm)2054  4.867673    0.655959   7.411 1.22e-05 ***
## as.factor(Cterm)2059  4.950178    0.656151   7.535 2.95e-05 ***
## as.factor(Cterm)2064  5.032683    0.656343   7.659 6.40e-05 ***
## as.factor(Cterm)2069  5.115188    0.656535   7.783 1.59e-05 ***
## as.factor(Cterm)2074  5.197693    0.656727   7.907 3.90e-05 ***
## as.factor(Cterm)2079  5.280198    0.656919   8.031 9.30e-05 ***
## as.factor(Cterm)2084  5.362703    0.657111   8.155 2.17e-05 ***
## as.factor(Cterm)2089  5.445208    0.657303   8.279 5.10e-05 ***
## as.factor(Cterm)2094  5.527713    0.657495   8.403 1.22e-05 ***
## as.factor(Cterm)2099  5.610218    0.657687   8.527 2.95e-05 ***
## as.factor(Cterm)2104  5.692723    0.657879   8.651 6.40e-05 ***
## as.factor(Cterm)2109  5.775228    0.658071   8.775 1.59e-05 ***
## as.factor(Cterm)2114  5.857733    0.658263   8.899 3.90e-05 ***
## as.factor(Cterm)2119  5.940238    0.658455   9.023 9.30e-05 ***
## as.factor(Cterm)2124  6.022743    0.658647   9.147 2.17e-05 ***
## as.factor(Cterm)2129  6.105248    0.658839   9.271 5.10e-05 ***
## as.factor(Cterm)2134  6.187753    0.659031   9.395 1.22e-05 ***
## as.factor(Cterm)2139  6.270258    0.659223   9.519 2.95e-05 ***
## as.factor(Cterm)2144  6.352763    0.659415   9.643 6.40e-05 ***
## as.factor(Cterm)2149  6.435268    0.659607   9.767 1.59e-05 ***
## as.factor(Cterm)2154  6.517773    0.659799   9.891 3.90e-05 ***
## as.factor(Cterm)2159  6.600278    0.659991   10.015 9.30e-05 ***
## as.factor(Cterm)2164  6.682783    0.660183   10.139 2.17e-05 ***
## as.factor(Cterm)2169  6.765288    0.660375   10.263 5.10e-05 ***
## as.factor(Cterm)2174  6.847793    0.660567   10.387 1.22e-05 ***
## as.factor(Cterm)2179  6.930298    0.660759   10.511 2.95e-05 ***
## as.factor(Cterm)2184  7.012803    0.660951   10.635 6.40e-05 ***
## as.factor(Cterm)2189  7.095308    0.661143   10.759 1.59e-05 ***
## as.factor(Cterm)2194  7.177813    0.661335   10.883 3.90e-05 ***
## as.factor(Cterm)2199  7.260318    0.661527   11.007 9.30e-05 ***
## as.factor(Cterm)2204  7.342823    0.661719   11.131 2.17e-05 ***
## as.factor(Cterm)2209  7.425328    0.661911   11.255 5.10e-05 ***
## as.factor(Cterm)2214  7.507833    0.662103   11.379 1.22e-05 ***
## as.factor(Cterm)2219  7.590338    0.662295   11.503 2.95e-05 ***
## as.factor(Cterm)2224  7.672843    0.662487   11.627 6.40e-05 ***
## as.factor(Cterm)2229  7.755348    0.662679   11.751 1.59e-05 ***
## as.factor(Cterm)2234  7.837853    0.662871   11.875 3.90e-05 ***
## as.factor(Cterm)2239  7.920358    0.663063   11.999 9.30e-05 ***
## as.factor(Cterm)2244  8.002863    0.663255   12.123 2.17e-05 ***
## as.factor(Cterm)2249  8.085368    0.663447   12.247 5.10e-05 ***
## as.factor(Cterm)2254  8.167873    0.663639   12.371 1.22e-05 ***
## as.factor(Cterm)2259  8.250378    0.663831   12.495 2.95e-05 ***
## as.factor(Cterm)2264  8.332883    0.664023   12.619 6.40e-05 ***
## as.factor(Cterm)2269  8.415388    0.664215   12.743 1.59e-05 ***
## as.factor(Cterm)2274  8.497893    0.664407   12.867 3.90e-05 ***
## as.factor(Cterm)2279  8.580398    0.664599   12.991 9.30e-05 ***
## as.factor(Cterm)2284  8.662903    0.664791   13.115 2.17e-05 ***
## as.factor(Cterm)2289  8.745408    0.664983   13.239 5.10e-05 ***
## as.factor(Cterm)2294  8.827913    0.665175   13.363 1.22e-05 ***
## as.factor(Cterm)2299  8.910418    0.665367   13.487 2.95e-05 ***
## as.factor(Cterm)2304  8.992923    0.665559   13.611 6.40e-05 ***
## as.factor(Cterm)2309  9.075428    0.665751   13.735 1.59e-05 ***
## as.factor(Cterm)2314  9.157933    0.665943   13.859 3.90e-05 ***
## as.factor(Cterm)2319  9.240438    0.666135   13.983 9.30e-05 ***
## as.factor(Cterm)2324  9.322943    0.666327   14.107 2.17e-05 ***
## as.factor(Cterm)2329  9.405448    0.666519   14.231 5.10e-05 ***
## as.factor(Cterm)2334  9.487953    0.666711   14.355 1.22e-05 ***
## as.factor(Cterm)2339  9.570458    0.666903   14.479 2.95e-05 ***
## as.factor(Cterm)2344  9.652963    0.667095   14.603 6.40e-05 ***
## as.factor(Cterm)2349  9.735468    0.667287   14.727 1.59e-05 ***
## as.factor(Cterm)2354  9.817973    0.667479   14.851 3.90e-05 ***
## as.factor(Cterm)2359  9.900478    0.667671   14.975 9.30e-05 ***
## as.factor(Cterm)2364  9.982983    0.667863   15.099 2.17e-05 ***
## as.factor(Cterm)2369  10.065488    0.668055   15.223 5.10e-05 ***
## as.factor(Cterm)2374  10.147993    0.668247   15.347 1.22e-05 ***
## as.factor(Cterm)2379  10.230498    0.668439   15.471 2.95e-05 ***
## as.factor(Cterm)2384  10.313003    0.668631   15.595 6.40e-05 ***
## as.factor(Cterm)2389  10.395508    0.668823   15.719 1.59e-05 ***
## as.factor(Cterm)2394  10.478013    0.669015   15.843 3.90e-05 ***
## as.factor(Cterm)2399  10.560518    0.669207   15.967 9.30e-05 ***
## as.factor(Cterm)2404  10.643023    0.669399   16.091 2.17e-05 ***
## as.factor(Cterm)2409  10.725528    0.669591   16.215 5.10e-05 ***
## as.factor(Cterm)2414  10.808033    0.669783   16.339 1.22e-05 ***
## as.factor(Cterm)2419  10.890538    0.669975   16.463 2.95e-05 ***
## as.factor(Cterm)2424  10.973043    0.670167   16.587 6.40e-05 ***
## as.factor(Cterm)2429  11.055548    0.670359   16.711 1.59e-05 ***
## as.factor(Cterm)2434  11.138053    0.670551   16.835 3.90e-05 ***
## as.factor(Cterm)2439  11.220558    0.670743   16.959 9.30e-05 ***
## as.factor(Cterm)2444  11.303063    0.670935   17.083 2.17e-05 ***
## as.factor(Cterm)2449  11.385568    0.671127   17.207 5.10e-05 ***
## as.factor(Cterm)2454  11.468073    0.671319   17.331 1.22e-05 ***
## as.factor(Cterm)2459  11.550578    0.671511   17.455 2.95e-05 ***
## as.factor(Cterm)2464  11.633083    0.671703   17.579 6.40e-05 ***
## as.factor(Cterm)2469  11.715588    0.671895   17.703 1.59e-05 ***
## as.factor(Cterm)2474  11.798093    0.672087   17.827 3.90e-05 ***
## as.factor(Cterm)2479  11.880598    0.672279   17.951 9.30e-05 ***
## as.factor(Cterm)2484  11.963103    0.672471   18.075 2.17e-05 ***
## as.factor(Cterm)2489  12.045608    0.672663   18.199 5.10e-05 ***
## as.factor(Cterm)2494  12.128113    0.672855   18.323 1.22e-05 ***
## as.factor(Cterm)2499  12.210618    0.673047   18.447 2.95e-05 ***
## as.factor(Cterm)2504  12.293123    0.673239   18.571 6.40e-05 ***
## as.factor(Cterm)2509  12.375628    0.673431   18.695 1.59e-05 ***
## as.factor(Cterm)2514  12.458133    0.673623   18.819 3.90e-05 ***
## as.factor(Cterm)2519  12.540638    0.673815   18.943 9.30e-05 ***
## as.factor(Cterm)2524  12.623143    0.674007   19.067 2.17e-05 ***
## as.factor(Cterm)2529  12.705648    0.674199   19.191 5.10e-05 ***
## as.factor(Cterm)2534  12.788153    0.674391   19.315 1.22e-05 ***
## as.factor(Cterm)2539  12.870658    0.674583   19.439 2.95e-05 ***
## as.factor(Cterm)2544  12.953163    0.674775   19.563 6.40e-05 ***
## as.factor(Cterm)2549  13.035668    0.674967   19.687 1.59e-05 ***
## as.factor(Cterm)2554  13.118173    0.675159   19.811 3.90e-05 ***
## as.factor(Cterm)2559  13.200678    0.675351   19.935 9.30e-05 ***
## as.factor(Cterm)2564  13.283183    0.675543   20.059 2.17e-05 ***
## as.factor(Cterm)2569  13.365688    0.675735   20.183 5.10e-05 ***
## as.factor(Cterm)2574  13.448193    0.675927   20.307 1.22e-05 ***
## as.factor(Cterm)2579  13.530698    0.676119   20.431 2.95e-05 ***
## as.factor(Cterm)2584  13.613203    0.676311   20.555 6.40e-05 ***
## as.factor(Cterm)2589  13.695708    0.676503   20.679 1.59e-05 ***
## as.factor(Cterm)2594  13.778213    0.676695   20.803 3.90e-05 ***
## as.factor(Cterm)2599  13.860718    0.676887   20.927 9.30e-05 ***
## as.factor(Cterm)2604  13.943223    0.677079   21.051 2.17e-05 ***
## as.factor(Cterm)2609  14.025728    0.677271   21.175 5.10e-05 ***
## as.factor(Cterm)2614  14.108233    0.677463   21.299 1.22e-05 ***
## as.factor(Cterm)2619  14.190738    0.677655   21.423 2.95e-05 ***
## as.factor(Cterm)2624  14.273243    0.677847   21.547 6.40e-05 ***
## as.factor(Cterm)2629  14.355748    0.678039   21.671 1.59e-05 ***
## as.factor(Cterm)2634  14.438253    0.678231   21.795 3.90e-05 ***
## as.factor(Cterm)2639  14.520758    0.678423   21.919 9.30e-05 ***
## as.factor(Cterm)2644  14.603263    0.678615   22.043 2.17e-05 ***
## as.factor(Cterm)2649  14.685768    0.678807   22.167 5.10e-05 ***
## as.factor(Cterm)2654  14.768273    0.678999   22.291 1.22e-05 ***
## as.factor(Cterm)2659  14.850778    0.679191   22.415 2.95e-05 ***
## as.factor(Cterm)2664  14.933283    0.679383   22.539 6.40e-05 ***
## as.factor(Cterm)2669  15.015788    0.679575   22.663 1.59e-05 ***
## as.factor(Cterm)2674  15.098293    0.679767   22.787 3.90e-05 ***
## as.factor(Cterm)2679  15.180798    0.679959   22.911 9.30e-05 ***
## as.factor(Cterm)2684  15.263303    0.680151   23.035 2.17e-05 ***
## as.factor(Cterm)2689  15.345808    0.680343   23.159 5.10e-05 ***
## as.factor(Cterm)2694  15.428313    0.680535   23.283 1.22e-05 ***
## as.factor(Cterm)2699  15.510818    0.680727   23.407 2.95e-05 ***
## as.factor(Cterm)2704  15.593323    0.680919   23.531 6.40e-05 ***
## as.factor(Cterm)2709  15.675828    0.681111   23.655 1.59e-05 ***
## as.factor(Cterm)2714  15.758333    0.681303   23.779 3.90e-05 ***
## as.factor(Cterm)2719  15.840838    0.681495   23.903 9.30e-05 ***
## as.factor(Cterm)2724  15.923343    0.681687   24.027 2.17e-05 ***
## as.factor(Cterm)2729  16.005848    0.681879   24.151 5.10e-05 ***
## as.factor(Cterm)2734  16.088353    0.682071   24.275 1.22e-05 ***
## as.factor(Cterm)2739  16.170858    0.682263   24.399 2.95e-05 ***
## as.factor(Cterm)2744  16.253363    0.682455   24.523 6.40e-05 ***
## as.factor(Cterm)2749  16.335868    0.682647   24.647 1.59e-05 ***
## as.factor(Cterm)2754  16.418373    0.682839   24.771 3.90e-05 ***
## as.factor(Cterm)2759  16.500878    0.683031   24.895 9.30e-05 ***
## as.factor(Cterm)2764  16.583383    0.683223   25.019 2.17e-05 ***
## as.factor(Cterm)2769  16.665888    0.683415   25.143 5.10e-05 ***
## as.factor(Cterm)2774  16.748393    0.683607   25.267 1.22e-05 ***
## as.factor(Cterm)2779  16.830898    0.683799   25.391 2.95e-05 ***
## as.factor(Cterm)2784  16.913403    0.683991   25.515 6.40e-05 ***
## as.factor(Cterm)2789  16.995908    0.684183   25.639 1.59e-05 ***
## as.factor(Cterm)2794  17.078413    0.684375   25.763 3.90e-05 ***
## as.factor(Cterm)2799  17.160918    0.684567   25.887 9.30e-05 ***
## as.factor(Cterm)2804  17.243423    0.684759   26.011 2.17e-05 ***
## as.factor(Cterm)2809  17.325928    0.684951   26.135 5.10e-05 ***
## as.factor(Cterm)2814  17.408433    0.685143   26.259 1.22e-05 ***
## as.factor(Cterm)2819  17.490938    0.685335   26.383 2.95e-05 ***
## as.factor(Cterm)2824  17.573443    0.685527   26.507 6.40e-05 ***
## as.factor(Cterm)2829  17.655948    0.685719   26.631 1.59e-05 ***
## as.factor(Cterm)2834  17.738453    0.685911   26.755 3.90e-05 ***
## as.factor(Cterm)2839  17.820958    0.686103   26.879 9.30e-05 ***
## as.factor(Cterm)2844  17.903463    0.686295   27.003 2.17e-05 ***
## as.factor(Cterm)2849  17.985968    0.686487   27.127 5.10e-05 ***
## as.factor(Cterm)2854  18.068473    0.686679   27.251 1.22e-05 ***
## as.factor(Cterm)2859  18.150978    0.686871   27.375 2.95e-05 ***
## as.factor(Cterm)2864  18.233483    0.687063   27.499 6.40e-05 ***
## as.factor(Cterm)2869  18.315988    0.687255   27.623 1.59e-05 ***
## as.factor(Cterm)2874  18.398493    0.687447   27.747 3.90e-05 ***
## as.factor(Cterm)2879  18.480998    0.687639   27.871 9.30e-05 ***
## as.factor(Cterm)2884  18.563503    0.687831   27.995 2.17e-05 ***
## as.factor(Cterm)2889  18.646008    0.688023   28.119 5.10e-05 ***
## as.factor(Cterm)2894  18.728513    0.688215   28.243 1.22e-05 ***
## as.factor(Cterm)2899  18.811018    0.688407   28.367 2.95e-05 ***
## as.factor(Cterm)2904  18.893523    0.688599   28.491 6.40e-05 ***
## as.factor(Cterm)2909  18.976028    0.688791   28.615 1.59e-05 ***
## as.factor(Cterm)2914  19.058533    0.688983   28.739 3.90e-05 ***
## as.factor(Cterm)2919  19.141038    0.689175   28.863 9.30e-05 ***
## as.factor(Cterm)2924  19.223543    0.689367   28.987 2.17e-05 ***
## as.factor(Cterm)2929  19.306048    0.689559   29.111 5.10e-05 ***
## as.factor(Cterm)2934  19.388553    0.689751   29.235 1.22e-05 ***
## as.factor(Cterm)2939  19.471058    0.689943   29.359 2.95e-05 ***
## as.factor(Cterm)2944  19.553563    0.690135   29.483 6.40e-05 ***
## as.factor(Cterm)2949  19.636068    0.690327   29.607 1.59e-05 ***
## as.factor(Cterm)2954  19.718573    0.690519   29.731 3.90e-05 ***
## as.factor(Cterm)2959  19.801078    0.690711   29.855 9.30e-05 ***
## as.factor(Cterm)2964  19.883583    0.690903   29.979 2.17e-05 ***
## as.factor(Cterm)2969  19.966088    0.691095   30.103 5.10e-05 ***
## as.factor(Cterm)2974  20.048593    0.691287   30.227 1.22e-05 ***
## as.factor(Cterm)2979  20.131098    0.691479   30.351 2.95e-05 ***
## as.factor(Cterm)2984  20.213603    0.691671   30.475 6.40e-05 ***
## as.factor(Cterm)2989  20.296108    0.691863   30.599 1.59e-05 ***
## as.factor(Cterm)2994  20.378613    0.692055   30.723 3.90e-05 ***
## as.factor(Cterm)2999  20.461118    0.692247   30.847 9.30e-05 ***
## as.factor(Cterm)3004  20.543623    0.692439   30.971 2.17e-05 ***
## as.factor(Cterm)3009  20.626128    0.692631   31.095 5.10e-05 ***
## as.factor(Cterm)3014  20.708633    0.692823   31.219 1.22e-05 ***
## as.factor(Cterm)3019  20.791138    0.693015   31.343 2.95e-05 ***
## as.factor(Cterm)3024  20.873643    0.693207   31.467 6.40e-05 ***
## as.factor(Cterm)3029  20.956148    0.693399   31.591 1.59e-05 ***
## as.factor(Cterm)3034  21.038653    0.693591   31.715 3.90e-05 ***
## as.factor(Cterm)3039  21.121158    0.693783   31.839 9.30e-05 ***
## as.factor(Cterm)
```

Some diagnostics: more work needed

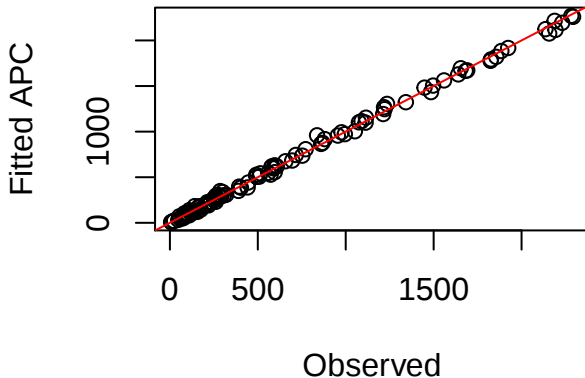
```
par(mfrow = c(1, 2))  
plot(fitted(glmMA) ~ dfEpi$D, ylab = "Fitted Age Only",  
     xlab = "Observed")  
abline(a = 0, b = 1, col = "red")
```



Fit is clearly inadequate

Some diagnostics: more work needed

```
plot(fitted(glmMACPexcess) ~ dfEpi$D, ylab = "Fitted APC",  
     xlab = "Observed")  
abline(a = 0, b = 1, col = "red")
```



Nothing jumps out...

Nielsen apc package

First get the data into the right format

```
library(apc)
resp <- matrix(dfEpi$D, nrow = 10, ncol = 11, byrow = F)
risk <- matrix(dfEpi$Y, nrow = 10, ncol = 11, byrow = F)
apc.data <- apc.data.list(resp, data.format = "AP",
  risk, age1 = 42.5, per1 = 1946.5, unit = 5)
```

Initial plot to summarize data

Note doses are the population at risk numbers.

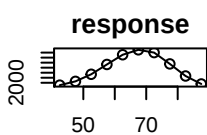
```
apc.plot.data.sums(apc.data)
```

This isn't as pretty as the Epi version.

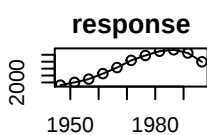
It produces marginal plots, i.e., numbers of cases (first row), population at risk (second row) and risk (third row), by age (first column), period (second column), cohort (third column).

Sums of data is a little confusing. . .

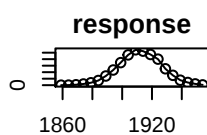
sums of data



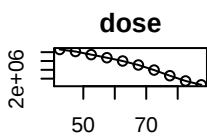
sums of data



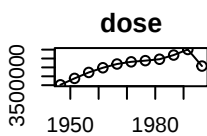
sums of data



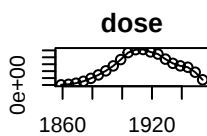
sums of data



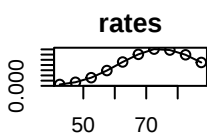
sums of data



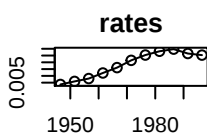
sums of data



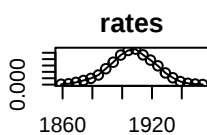
sums of data



sums of data



sums of data

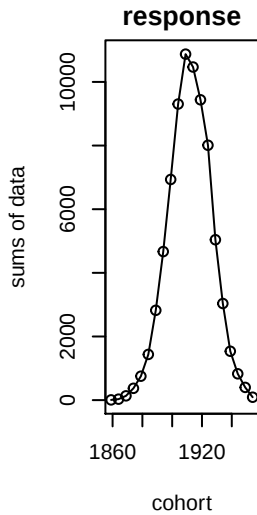
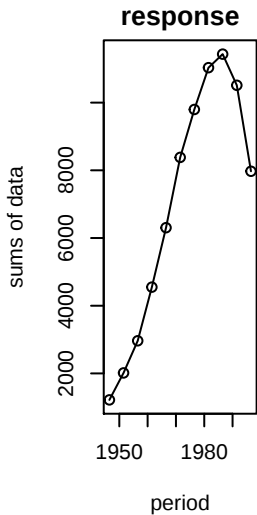
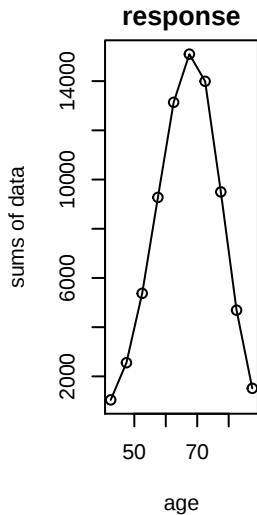


cohort

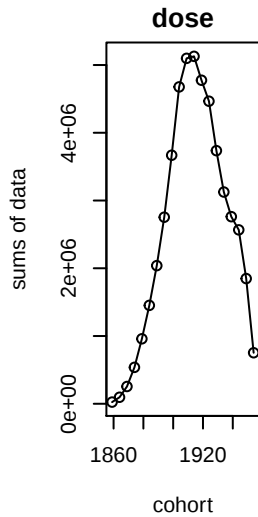
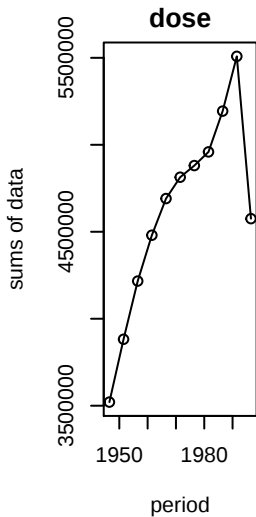
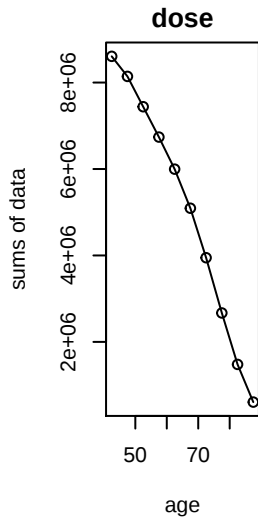
period

age

The counts only



The populations at risk only



The risks only

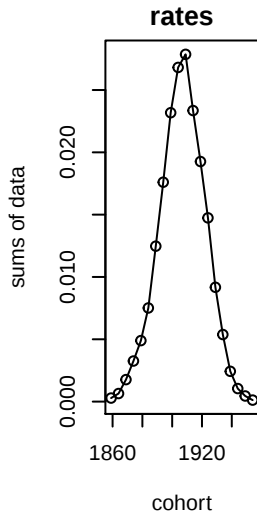
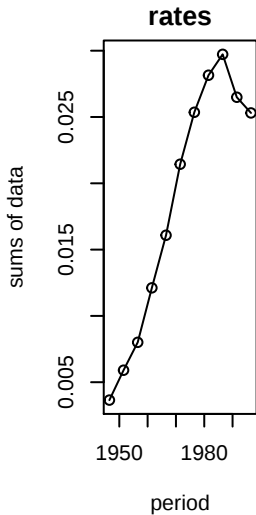
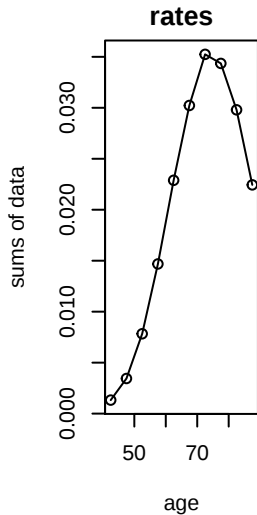


Table of deviances - agrees with Epi package!

Canonical parameterization of Kuang et al (2008)

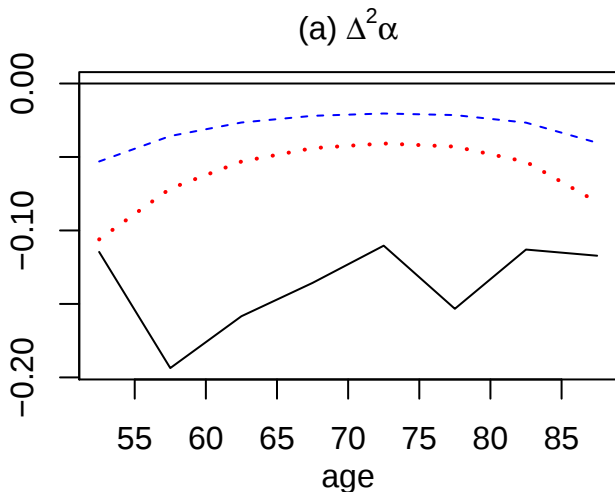
```
apc.dev <- apc.fit.table(apc.data, "poisson.dose.response")
```

```
apc.dev[1:15, c(1, 2, 3, 4, 5, 6)]
```

##	-2logL	df.residual	prob(>chi_sq)	LR.vs.APC	df.vs.APC	prob(>chi_sq)
## APC	208.548	72	0	NA	NA	NA
## AP	2723.466	90	0	2514.918	18	0
## AC	829.629	81	0	621.082	9	0
## PC	8609.929	80	0	8401.381	8	0
## Ad	6417.381	99	0	6208.833	27	0
## Pd	15470.469	98	0	15261.922	26	0
## Cd	9434.668	89	0	9226.121	17	0
## A	15103.001	100	0	14894.454	28	0
## P	56122.067	99	0	55913.520	27	0
## C	55099.747	90	0	54891.199	18	0
## t	19656.557	107	0	19448.010	35	0
## tA	27614.467	108	0	27405.919	36	0
## tP	61251.648	108	0	61043.100	36	0
## tC	68874.518	108	0	68665.970	36	0
## 1	71776.177	109	0	71567.629	37	0

Second differences for age: 1/2 SE bands in blue/red

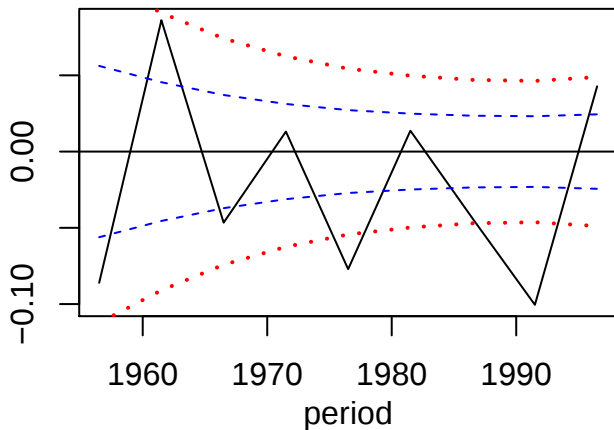
```
fit.apc <- apc.fit.model(apc.data, "poisson.dose.response",  
  "APC")  
apc.plot.fit(fit.apc, sub.plot = "a")
```



Second differences for period

```
apc.plot.fit(fit.apc, subplot = "b")
```

(b) $\Delta^2\beta$



Second differences for cohort

```
apc.plot.fit(fit.apc, subplot = "c")
```

(c) $\Delta^2\gamma$

