Proceedings of the ASME 2019 38th International Conference on Ocean, Offshore and Arctic Engineering
OMAE2019
June 9-14, 2019, Glasgow, Scotland

OMAE2019-96802

A PREILIMINARY STUDY ON THE MODELING AND ANALYSIS OF NONLINEAR EFFECTS OF OCEAN WAVES AND POWER-TAKE-OFF CONTROL ON WAVE ENERGY CONVERSION SYSTEM DYNAMICS

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ABSTRACT

This article describes the model development and preliminary progress of an on-going research study on the effects of nonlinearities in ocean wave input and power-take-off (PTO) control on wave energy conversion system dynamics and efficiency. The model system employed and progress on recent developments are: (1) nonlinear wave modeling in the ocean, generation and propagation in a wave basin, and (2) nonlinear PTO control algorithm. An overview of the holistic analytical, numerical and experimental research approach/work plan is presented. To provide a simple means for analysis, comparison and performance evaluation, the WEC-Sim numerical platform is used for model implementation and system dynamic simulation. Analytical and numerical predictions of the nonlinear wave fields in a wave basin using the nonlinear Fourier analysis (NLFA) technique and corresponding nonlinear wavemaker theory and a plan for future validation using a comprehensive series of experimental test data as well as ocean wave measurements are described. Efficiency of the nonlinear PTO control and a future evaluation work plan by comparing numerical simulations with results of WEC model test data under corresponding wave conditions of the experimental studies without the presence of the WEC system are also presented.

Keywords: Nonlinearity, ocean wave models, wave energy conversion, power-take-off control, wave basin experiments, and field data.

1 INTRODUCTION

Marine hydrokinetics in general and wave energy conversion (WEC) in particular have been topics of interest since the 1970's and much more intensely studied for the last couple of decades. With the support of the US Department of Energy (DOE), personnel at the Pacific Marine Energy Center (PMEC) have been serving the hydrokinetic research communities since 2008 by conducting research as well as supporting large-scale experiments conducted in the wave basins by researchers around the country. Research projects conducted at PMEC including environmental conditions, WEC hydrodynamics, power-take-off (PTO) modeling, analysis and experimental verification. A significant number of the projects centers on the development of WEC-Sim, a community software based on relatively simple analysis models for preliminary design of WEC systems.

At PMEC, one of our recently initiated projects focuses on the effects of nonlinearity on efficiency of wave energy conversion systems. A number of the dominant nonlinear hydrodynamic effects such as nonlinear Froude-Krylov and viscous forces, and nonlinear effects in the power takeoff (PTO) systems such as friction, valve characteristics, fluid compressibility, Wheeler stretching at the free surface, nonlinear mooring lines, and nonlinear fluid-structure interaction effects, etc., over and above the linear analysis already in WEC-Sim have either already been incorporated or being developed by WEC-Sim developers and researchers. The focus of our study is to further enhance the WEC-Sim modeling capability in power production mode under mild nonlinear seas, specifically on the following two nonlinearities: (1) modeling of the wave field under moderate and extreme conditions and (2) modeling nonlinear power-take-off control systems using a fuzzy control strategy. Second- and third-order nonlinear wave theories and numerical algorithms will be developed to more accurately model the wave excitation forces on the WEC system. This paper describes a roadmap for achieving the objectives in the next few years. In the project, advanced nonlinear PTO control algorithms will be developed. The resulting nonlinear wave input and PTO control models will be validated first against existing experimental wave basin nonlinear tests and representative WEC system test data at the OSU Hinsdale Wave Research Laboratory and ocean wave measurements from the SWIFT buoys.

Additional nonlinear wave model and WEC system tests will be conducted to complement the existing data and fill in the gaps needed to thoroughly validate the advanced nonlinear models developed in this study. To make the project grounded in application and for ease of understanding and reference, we will tie the applications of our study to the open-source community model WEC-Sim.

The contents of the rest of the article are as follows. First we review literature on nonlinear ocean wave models including the nonlinear Schrodinger equation and associated inverse scattering transform analytical solution and the higher-order spectral method in Section 2. Because the new developments will be integrated into WEC-Sim, a brief review of the internal logic and algorithms of the open-source code is provided in Section 3. We then present the ideas on nonlinear ocean wave models and nonlinear PTO control theory and algorithms including reinforcement learning and fuzzy logic in Sections 4 and 5, respectively. Existing wave, WEC and PTO control basin test data and additional new tests to be conducted are described in Section 6. The available measured ocean wave data from the SWIFT buoys that will be used to validate the nonlinear wave and PTO models is summarized in Section 7. Finally, concluding remarks on future plans of the project are provided in Section 8.

2 LITERATURE REVIEW

The study of water wave behavior is perhaps one of the oldest research topics in engineering. More complicated computational fluid dynamics (CFD) models and large-scale experimental facilities have been developed, along the advancements in computing resources, mathematical developments, and with increasing demand, especially from the Energy industry (especially oil and hydrokinetics). In comparison, the trend in improvement and developments have been much slower for analytical fluid dynamics (AFD), due to lack of efficient mathematical tools. Also, highly demanding mathematical subjects that are usually involved in AFD, propels researchers to take alternative approaches such as computational fluid dynamics (CFD). This article first outlines the current status of research and application of wave field modeling, analysis, and generation, to pave the way to identifying the gaps and improvement opportunities.

The nonlinear Schrödinger (NLS) equation was originally derived from the Zakharov equations [1] and it describes the 1+1D wave dynamics in deep-water conditions. The equation is derived through the expansion of the frequencies about a carrier frequency to the second order and replacing the interaction term with its value at the carrier frequency.

From a wave analysis point of view, the goal of analyzing wave data is to understand the causal loop in which the complex hydrodynamic behavior of a wave field forms and is observed. One of the most intuitive approach is to decompose the wave train into some form of components, which in return would simplify the problem in hand. There are many ways to perform such transformation, with both linear and nonlinear assumptions, or different properties and domains of transformation. The domain of transformation is the space on which the transformed time (space)-series lie in, for example a frequency (wavenumber) or a time (space)-frequency (wavenumber) domain. Also, the concept of linearity and nonlinearity are different in data analysis than in a wave equation. The most straight forward definition of linearity in a transformation is first, if all the input is multiplied by some factor, then the transformed components are amplified with the same factor. Second, the superposition is allowed for the decomposed components. If a transformation does not have any of these two characteristics, then it should be considered a nonlinear transformation.

For simplicity, and the fact that wave data are usually available in the form of time series, the types of corresponding analysis domains would be frequency and time-frequency domains. Time information of a frequency domain analysis is lost and the resulting spectrum provides information about the amplitude or energy distribution over a range of frequencies. Since all the information about time is now disregarded, hence, the frequencies are assumed to be stationary and in permanent form, which would oscillate infinitely, infinite being a measure of duration of the recorded time series. In a time-frequency analysis the time information of each frequency component is being preserved, so each may behave in a stationary or nonstationary manner. Among some of the most implemented transformations are the linear (fast) Fourier (FFT), Hilbert-Huang (HHT), and inverse scattering (IST) transformations. Since the focus of this study is the implementation of nonlinear wave field approximations in WEC-Sim, the analytic IST transformation is adopted here.

Computational fluid dynamics (CFD) is one of the most commonly approaches in modeling wave fields, both in open ocean and closed basins. CFD owes its popularity to the ease of use and usually user friendly interfaces, along with the vast amount of output information provided after each simulation. The most well-known CFD codes are the Navier-Stokes (NS) solvers, which are usually based on one of finite difference (FD), finite volume (FV), or finite element (FE) methods. The main drawback of using a NS solver is the required computing time, which usually takes in order of days and weeks for a thirty minutes simulation time. For most of the engineering applications, assuming an incompressible, irrotational, and inviscid flow is an accurate assumption and such assumptions guarantees the existence of a velocity potential and reduces the NS equations to Euler equations. Potential flow solvers are much faster in comparison to the NS solvers and most of them are based on a boundary element or higher order spectral methods. A review of a higher order spectral method solver developed by LHEEA-ECN (Hydrodynamics, Energetics and Atmospheric Environment Laboratory of Ecole Centrale de Nantes) group to be implemented in the present study in validation and generation of the wave environment is provided in the following paragraph.

The High-Order Spectral method (HOSM) includes two packages, namely HOS-NWT, a numerical wave tank model with wavemaking and absorption capabilities, and HOS-Ocean, an open boundary solver to approximate wave field transformation from an initial state. HOS method owes its accuracy and efficiency to the pseudo-spectral approach in the solution procedure and has been proven to be more efficient than finite-base discretization models [2], [3]. The solution procedure details and formulation of the problem can be found in the original works of [4] and [5]. In recent years, there have been an increase in implementation of HOS packages that provided a large database of validation available and the code can be considered mature for engineering practices [3]. Some of the applications that could be more appealing to the energy industry and deep-water wave modeling are, but not limited to, modulational instabilities ([6],[7]) and freak waves ([8], [9], [10]). It should be noted that HOSM is a strong nonlinear model and can include the wave-wave interactions up to a defined order, which is called the order of HOSM. For instance with an order of 3 HOSM model, an accuracy equal to that of Zakharov equation [1] can be achieved [9]. Further investigation revealed that an order 6 HOSM model with formulation of [5] corresponds to a nearly fully nonlinear model [11].

One of the project goals is to explore and develop nonlinear PTO control approaches that not only improves power capture compared to simple, traditional linear approaches (e.g., damping), but also satisfies additional considerations of ease of implementation and application by engineers without a theoretical control background, and can be easily implemented in WEC-Sim. A survey of candidate PTO control approaches has been conducted, including linearization techniques, phase plane (topological) control, describing functions, reinforcement learning, and fuzzy control. Of these candidates, reinforcement learning and fuzzy control have been selected for further development for inclusion in WEC-Sim.

WEC-Sim is an established wave energy modeling tool with particular application to early stage WEC researchers and developers in need of validating their design prior to significant investment in physical modeling. As developers advance in TRL, a linear modeling approach may no longer be appropriate for the needs of this focus group. WEC-Sim provides a relatively friendly workflow and relatively low-cost alternative to commercial WEC modeling packages. It provides a six-degree of freedom time-domain simulation environment created explicitly for WEC design and evaluation. The code is open source and developed in the MATLAB/Simulink environment. MATLAB, Simulink, Simscape, and Simscape Multibody are required to run the basic configuration of WEC-Sim. Additional toolboxes may be necessary for more complex modeling scenarios.

The analytical and numerical models described herein are mathematical representations of propagating water waves. In this context, an analytical model is obtained after the integration of the PDE system, where a series of assumptions and simplifications have been undertaken, and more importantly, the equations have been linearized in such a way that a continuous solution of the unknowns (e.g. the surface elevation, pressure and/or velocity field) is obtained. Moreover, analytical solutions to the nonlinear wave propagation problem have been found applying mathematical techniques such as perturbation series, where the nonlinearity is considered weak for the series to be convergent.

On the other hand, numerical models are those where the equations of motion are solved by applying numerical integration techniques, which implies that the governing equations have been discretized, and a solution to the unknowns is obtained with the same spatial or temporal discretization. The primitive governing equations (as well as the boundary conditions) can be as complex and nonlinear as required. However, domain size and computational time are still the major limitations to this approach. The analytical and numerical models presented in the previous sections are some of the multiple versions of different models solving the propagation of water waves.

Regardless of the complexity of the model, as indicated above, some assumptions and simplifications were taken. Even for the most general expressions, mass and momentum conservation is also an assumption. Therefore, measurements and data comparison are required for model calibration and validation. Laboratory data is, in general, the preferred source for model validation since boundary and initial conditions are controlled, the repeatability and, hence, the uncertainty of the measurement can be assessed, and the conditions can be considered deterministic, i.e. the wave conditions are pre-defined.

In the context of physical model testing, experiments carried out in the absence of any specimen, model, structure or device, are defined as undisturbed wave tests, in the sense that the waves propagate without being disturbed by the device. Also known as wave calibration tests, these tests are the ones selected to perform analytical and numerical model validation of the wave propagation. Once the mathematical models have been validated, they can be used in the presence of physical models, e.g. wave energy converters, where the response of the device is investigated.

Field data are available for validation of time series generation and wave maker theory. The data were collected offshore of Oregon (USA) during six extreme wave events with significant wave heights exceeding 8 m [13]. Data collection used SWIFT buoys, which employ high sampling rates to capture wave breaking and other dynamics process [14]. The existing field data area well-suited to this project because 1) the wave conditions are likely to have strong nonlinearities, and 2) the buoys were deployed in pairs separated by a few wavelengths, such that coherent wave predictions can be tested.

3 WEC-SIM SYSTEM OVERVIEW

As the users of WEC-Sim add complexity to their model, modeling nonlinear components of the simulation become more attractive. The WEC-Sim developers have already implemented non-linear hydrostatic restoring and Froude-Krylov forces as an option. They also suggest other parts of the model where nonlinearities could be introduced.

Non-linear hydrostatic restoring and Froude-Krylov forces are particularly appropriate for bodies where the wetted surface changes with time. WEC-Sim can, in each time step, recalculate the wetted surface and the corresponding hydrostatic and Froude-Krylov force to be applied.

Other body related nonlinear components which can be applied in WEC-Sim are a quadratic drag force and Morison element terms. The values for these parameters are typically heuristic and determined experimentally, through free decay and forced oscillation tests. Additionally, nonlinear PTO and mooring systems can be implemented in WEC-Sim. There are non-linear elements in the PTO-Sim module which can be applied.

Fundamentally, WEC-Sim solves an equation of motion in six-degrees of freedom as follows:

$$m\ddot{X} = F_{exc}(t) + F_{rad}(t) + F_{PTO}(t) + F_v(t) + F_{ME}(t) + F_B(t) + F_m(t) \quad (1)$$

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where m is the mass matrix, $\ddot{X}$ is the acceleration vector of the device, $F_{exc}(t)$ is the wave excitation force and torque, $F_{rad}(t)$ is the force and torque vector resulting from wave radiation, $F_{PTO}(t)$ is the power take off force and torque vectors, F_v is the viscous force terms, $F_{ME}(t)$ is the Morison element force and torque vector, $F_B(t)$ is the net buoyancy restoring force and torque vector, and F_m is the force and torque vector resulting from the mooring connection. Implementation of nonlinearities can be realized in any of these terms to improve their prediction of a real-system analysis.

The focus of this project will be on improving the capabilities of WEC-Sim to include more nonlinear capabilities. Primary areas of contribution include nonlinear wave input, mooring response, and PTO control as addressed in other sections of this paper. The plan is to integrate these enhancements into the main WEC-Sim distribution, following the system set up by the WEC-Sim code developers. The code is hosted on GitHub, and a process of forking a version of the code and then submitting a pull request to the creators will be taken. Wave generation enhancements could potentially be added directly to the wave class, whereas mooring and control additions could be added as WEC-Sim modules. Alternatively, if deemed necessary, code written in another programming language such as C++ or Python could be coupled to WEC-Sim. An example of this is the MoorDyn mooring simulator (written in C++) which couples with WEC-Sim.

WEC-Sim is a powerful modeling tool for WEC researchers and developers. Although appropriate as is for most applications, the code could benefit from more non-linear treatment of many aspects of the code. Efforts will be focused on improving the non-linear implementation of input wave conditions, mooring analysis, and control approaches.

In general, considering three steps of WEC-Sim operation, namely preprocessing, time domain analysis, and the post processing and visualization, only the improvement in the frame of time domain analysis are included in the current study. To this end, some necessary details of the analysis procedure is presented here. All the steps of the analysis are considered based on the linear hydrodynamics coefficients and linear analysis except for the following parts. A weekly nonlinear approach [15] is adopted to capture above still water level, represented by D, wave properties, which is ignored by linear wave theory, by starching the z-coordinate to the instantaneous water level as:

$$z^* = \frac{D(D+z)}{D+\eta} - D \quad (2)$$

Also, the mooring force computations are nonlinear, considering the implementation of MoorDyn package in WEC-Sim. So, it can be seen that the potential steps for improving the WEC-Sim computation abilities are lying in the wave field analysis and PTO control systems. Although a major improvement can be performed on the preprocessing step of computing the hydrodynamic coefficients using a BEM solver, but this is out of scope of this research.

The wave field analysis currently used in WEC-Sim is based on linear wave theory, including the computations of free surface elevation and orbital velocities. This fact restricts the implementation of WEC-Sim to relatively low sea states and ignores the nonlinearities in wave profile, instabilities in deep water waves, and interactions between wave components. Since the WECs are designed to operate in moderate and higher sea states and to survive extreme sea conditions, such limitation does not agree with the expectations. An important improvement is to replace the current linear wave theory computations with a nonlinear approximation of the wave field. This nonlinear wave model can be either from an analytical solution of nonlinear wave equation, for example nonlinear Fourier analysis of the nonlinear Schrödinger equation, or results from a CFD code.

4 NONLINEAR OCEAN WAVE MODELS

The “art” of wave field modeling starts with setting up the original initial boundary value problem (IBVP) of a fluid domain, here for example is a closed basin of a wave experimental facility. The IBVP consists of a set of field equations that are usually based on conservation laws, governing the inside of the fluid domain, and a set of boundary conditions. Figure 1 presents an example of such an IBVP, formulated for an incompressible, irrotational, and inviscid fluid.

The investigation and understanding of any wave field behavior is through the solution of the IBVP depicted in Figure 1. The approach to understand such a complex system needs additional tools including simplifying assumptions, numerical approximations, and perhaps some solution domain transformations. Depending on the decided approach, different wave modeling procedures are available, ranging from complete Navier-Stokes numerical solvers to analytical linear wave theory with spectral methods. CFD provides a numerical approximation and solution of the original or modified IBVP and AFD solves the wave equations by providing analytical solutions, if possible, to the IBVP.

The most general form of field equations for explaining the behavior of fluid, assuming it is Newtonian, is the Navier-Stokes (NS) equations. The NS equations are one of the most comprehensive systems of equations for fluid dynamics, consisting of three scalar momentum equations, governing the conservation of momentum, in three-dimensional Cartesian coordinates, as presented in Equations 3. These equations are always solved, implicitly or explicitly, together with the conservation of mass equation (Eq. 4).

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}.\nabla)\boldsymbol{u}\right) = -\nabla p + \nabla.\left(\mu(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T) - \frac{2}{3}\mu(\nabla.\boldsymbol{u})\boldsymbol{I}\right) + \boldsymbol{F} \quad (3)$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \boldsymbol{u}) = 0 \quad (4)$$

in which, $\boldsymbol{u} = (u, v, w)$ is the velocity vector, p is the pressure, ρ represents the fluid density, μ is the dynamic viscosity of the fluid, $\boldsymbol{I}$ represents an identity matrix, and $\boldsymbol{F}$ is the body forces [18].

The derivation procedure and more details can be found in many standard hydrodynamics or water wave mechanics books. The reader is encouraged to find details especially in [16], [17], and [18] among others.

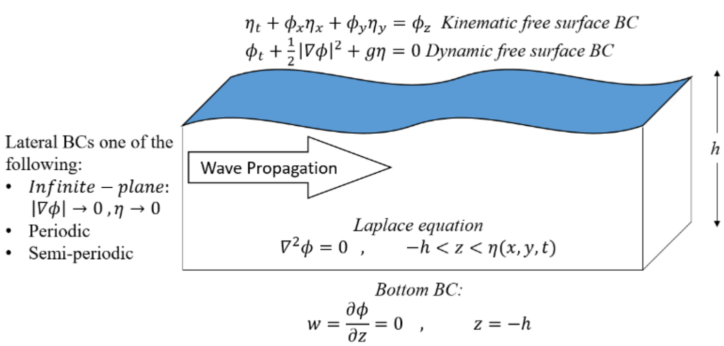

Figure 1: IBVP of a fluid domain.

In the present research, the main focus is on non-breaking, incompressible, irrotational, and inviscid wave fields. Such assumptions guarantees the existence of a velocity potential, $\phi(x, y, z, t)$, which provides information on the fluid particle velocity vector as,

$$\boldsymbol{u} = \nabla\phi \quad (5)$$

where $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$. The surface tension effect is assumed to be negligible and the depth is constant at $z = -h$. The wave profile, free-surface elevation is represented by $z = \eta(x, y, t)$. Under these assumptions, the NS equations are reduced to Euler equations ([16], [19]), as

$$\nabla^2\phi = 0, \ -h < z < \eta(x, y, t) \quad (6)$$

$$\frac{\partial \phi}{\partial z} = 0, \ z = -h \quad (7)$$

$$\eta_t + \phi_x\eta_x + \phi_y\eta_y = \phi_z, \ z = \eta(x, y, t) \quad (8)$$

$$\phi_t + \frac{1}{2}|\nabla\phi|^2 + g\eta = 0, \ z = \eta(x, y, t) \quad (9)$$

As can be seen from the Euler equations, Equations 6-9, the source of nonlinearity of the wave problem is in the free surface. On this boundary, there are nonlinear boundary conditions in addition to the unknown location of the free surface. Additionally, for the general IBVP of the fluid domain, additional lateral boundary conditions are needed to define the wave field dynamics. Such boundary conditions can be defined in two ways, (1) through an infinite-plane assumption, Equation 10, or (2) a periodic boundary condition [20], Equation 11.

$$|\nabla\phi| \to 0, \eta \to 0 \text{ as } (x^2 + y^2) \to \infty \quad (10)$$

$$\eta(x + L_x, y + L_y, t) = \eta(x, y, t)$$
$$\phi(x + L_x, y + L_y, t) = \phi(x, y, t) \quad (11)$$

The Euler equations are a good starting point for derivation and explanation of the wave equations, but they can only be solved using numerical approximations using CFD. Some of the most appropriate numerical tools to approximate and solve the Euler equations are higher order spectral methods (HOSM) and boundary element methods (BEM). The Euler equations can be used in deriving approximate wave equations by method of multiple scales [19]. Such wave equations can be the doorways to analytical solutions of the wave field and can provide in depth understanding of the underlying physical phenomenon.

The motion and behavior of water waves, in addition to the general IBVP, can also be explained using wave equations. These wave equations, which mostly govern the motion of the free surface, are usually derived from simplification and perturbation of the wave IBVP. Wave equations come in the form of linear or nonlinear equations, depending on the derivation procedure, and explain different aspects of the wave behavior, such as nonlinearity and dispersion. Most of the nonlinear wave equations are derived by nonlinear singular perturbation of the Euler equations [19]. In return, if the perturbation, or the nonlinearity, parameter is allowed to be small enough, then the linear version of the nonlinear equations are recovered. It should be mentioned that all the waves in this study are assumed to propagate in 1+1D domain, which is 1-dimensional in space and time.

4.1 Nonlinear Schrödinger Model for Deepwater

The NLS equation, for deep-water narrow-banded wave fields, is given as ([22], [23]):

$$i(\psi_t + C_g\psi_x) + \mu\psi_{xx} + \nu|\psi|^2\psi = 0 \quad (12)$$

in which, ψ is the complex envelope function of the wave train, $C_g = \omega_0/2k_0$ is the deep water group velocity, $\mu = -\omega_0/8k_0^2$ represent the dispersivity coefficient, $\nu = -\omega_0 k_0^2/2$ is the nonlinearity coefficient, and ω_0 and k_0 are the carrier circular frequency and wave numbers. From a more physical point of view, the NLS equation presents a balance between dispersive and nonlinear effects in a wave field, which are the second and the third term in Equation 10, respectively. This form of the NLS is called space-like NLS, sNLS [23], and is suitable for initial value problems (Cauchy problems) where the initial wave field as a function of spatial coordinates is known and the question is about the transformation of this initial profile in the space-time domain. The boundary value problem with measured free surface elevation time series at a fixed location can be analyzed and modeled using the time-like nonlinear Schrödinger, tNLS, as [23]:

$$i(\psi_x + C_g'\psi_t) + \mu'\psi_{tt} + \nu'|\psi|^2\psi = 0 \tag{13}$$

in which, $C_g' = 1/C_g$, $\mu' = \mu/C_g^3$, $\nu' = \nu/C_g$, and $\rho' = \rho C_g$. In general, the solutions of sNLS are related to those from tNLS through the following simple transformations:

$$x \to t\,, t \to x\,, \rho \to \rho'\,, \nu \to \nu', \mu \to \mu' \tag{14}$$

The amplitude of the free surface is given by [23]:

$$\eta(x,t) = \psi(x,t)e^{ik_0x - i\omega_0 t} + c.c. \tag{15}$$

in which $c.c.$ is the complex conjugate. One of the practical and interesting properties of the solutions of the NLS equation is that they contain the small-modulation instability, known as the Benjamin-Feir instability (BFI) [24]. This instability phenomenon is one of the important characteristics of deep-water waves and as waves propagate from shallower regions, depth effects control such instabilities. These unstable solutions of the NLS are called "breathers", which are the sources of extreme, or rogue, wave formation. Considering NLS equation in analysis and modeling of wave field for WEC design could improve the survivability of the deployed devices. Note that the BFI is weakly stable for waves with increasing directional spreading.

4.2 Nonlinear Fourier Analysis of NLS

The nonlinear Fourier transformation (NLFT) or inverse scattering transformation (IST) is a mathematical tool to solve integrable nonlinear partial differential equations (PDEs), some of which are the nonlinear wave equations of KdV, KP, NLS, etc. The general outline of NLFT is described in [23], [25], and [26], some details of which is presented here.

The first step of NLFT is to assume a PDE that best explains the considered physical phenomenon. Next, using the Lax pairs of the assumed PDE, the eigenvalue problem (specifically, the Zakharov-Shabat eigenvalue problem [27]) is constructed. Solution of the eigenvalue problem provides the nonlinear spectrum components, which is called direct NLFT or direct scattering transformation (DST). Finally, using the nonlinear spectrum values, the inverse problem can be solved using the superposition of nonlinear wave components and their interactions. Readers are encouraged to read the details of computing steps in [23] and for an extensive reference listing. The problem outlines here are based on a Cauchy approach (an initial value problem with space-series) but can be easily extended to boundary value problem solution (with measured time-series) using the transformations outlined in this section.

The focus of this research is on nonlinear analysis and modeling of WEC operational wave environment, which can be assumed to be mostly in deep-water, so the nonlinear Schrödinger equation (NLS) is chosen as the considered wave equation, the details of which is presented in the previous sections. Using the following transformations:

$$u = \tilde{\rho}\psi, T = \mu t, X = x - C_g t \tag{16}$$

the space-like Schrödinger equation (sNLS) becomes

$$iu_T + u_{XX} + 2|u|^2u = 0 \tag{17}$$

in which $u(x,t)$ is the input of the NLFT, and is the scaled dimensional complex envelop scaled by a nonlinearity parameter, $\tilde{\rho}$, explained in the previous sections. The corresponding eigenvalue problem of sNLS becomes [23],

$$\Psi_x = Q(\lambda)\Psi\ , \qquad Q = \begin{pmatrix} -i\lambda & u \\ -u^* & i\lambda \end{pmatrix} \tag{18}$$

where λ is the time independent complex eigenvalue. The most general solutions for NLS are the ones given by the inverse scattering transform (IST) [23]. The results from NLFT states that wave trains consist of a linear superposition of sine waves, Stokes waves, and breather trains, plus nonlinear pairwise interactions among these components [28]. The mathematics is a kind of nonlinear superposition principle which is constructed as the general nonlinear spectral solution of the nonlinear Schrödinger equation and associated Riemann theta functions [23]. In this formulation, the nonlinear spectrum is a complex period matrix in which the diagonal elements correspond to Stokes waves, and the off-diagonal elements indicate the strength of the nonlinear interactions between the components [28].

Implementing NLFA to compute the solutions of nonlinear wave equations automatically contains all free and bound modes of the solution. Sine waves and Stokes waves are presenting free and bound modes, respectively. The most important advantage of NLFA is an analytical representation for all nonlinear Fourier components. The components in NLFA, a nonlinear spectral (Fourier) theory, include sine waves, Stokes waves, phase locked Stokes waves known as breathers and solitons.

The nonlinear spectrum resulting from NLS equation, for examples see [23][25][26][28], consists of two main parts, the main spectrum, and spines. The stability of each of the wave frequency components can be determined from the spine formation. If the spine is a line going through the real axis (frequency axis), then this mode is a stable Stokes wave. If the spine connects two points of the spectrum and never crosses the real axis, then the resulting mode is an unstable breather packet, or equivalently a pair of phased-locked Stokes waves. Some of the most recent application of nonlinear spectrum in energy computation of wave trains and approximation of the extreme wave conditions can be found in [25] and [26].

4.3 Higher-Order Spectral Ocean Wave Model

In the potential flow assumption, the field equation is the Laplace equation in 3-D as:

$$\nabla_2^2\phi + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{19}$$

in which, ∇_2 is the Laplacian in 2D in the horizontal directions. The free surface boundary, both kinematic and dynamic, are written in term of the velocity potential on the free surface, $\tilde{\phi}(x,t) = \phi\big(x, z = \eta(x,t)\big)$, following [1] as:

$$\frac{\partial \eta}{\partial t} = (1 + |\nabla_2 \eta|^2)W - \nabla_2 \tilde{\phi}.\nabla_2 \eta \ ,$$
$$W = \frac{\partial \phi}{\partial z}(x, z = \eta, t) \qquad (20)$$
$$\frac{\partial \tilde{\phi}}{\partial t} = -g\eta - \frac{1}{2}\left|\nabla_2 \tilde{\phi}\right|^2 + \frac{1}{2}(1 + |\nabla_2 \eta|^2)W^2 \qquad (21)$$

in which W is the vertical velocity at the free surface and is the only variable that needs solution in the fluid domain.

In pseudo-spectral method of solution, computation of some of the equations, usually including products of variables, are performed in the physical domain and the rest, usually including the derivatives of the variables, in the Fourier space. This approach exhibits some amazing convergence properties. The addition of a second-order wavemaker theory in the HOS-NWT makes this package even more suitable for validation and prediction of the wave conditions. Details of the additional wavemaker boundary condition and the solution procedure is well defined in [2].

HOS-NWT was validated by selected experimental data obtained from Oregon State University O.H. Hinsdale Wave Research Laboratory. Figure 2 presents the comparison between the measured free surface elevation and the predictions from HOS-NWT at different wave gauge locations. An overall good agreement can be seen between the predicted and measured values which provides a better confidence in implementation of HOSM for prediction of wave field characteristics and elevation/orbital velocity time series. More detail on the computational steps and details of discretization is not presented here and the reader is encouraged to find such information following [3] and other publication referenced in this section.

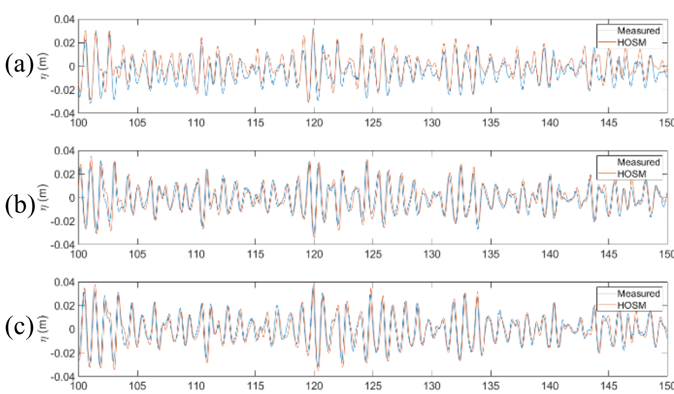

Figure 2: An example of HOSM validation vs. the experimental results for JONSWAP waves with $H_s = 0.045\ m$, $T_p = 1.0\ s$, and (a) $\gamma = 3.3$, (b) $\gamma = 5.0$, (c) $\gamma = 7$.

5 NONLINEAR PTO CONTROL ALGORITHMS

5.1 Reinforcement Learning

Assume a PTO control law of

$$F_{pto} = c\,\dot{z} + k\,z \qquad (22)$$

where c is the PTO damping, k is the PTO stiffness, and $\dot{z}$ and z are the WEC speed and position, respectively. The application of reinforcement learning is the online tuning of the damping and stiffness to maximize power production (or whatever the designer may wish to include in the objective function, for example, power could be balanced against operation near or over the systems constraints to avoid damage and wear-and-tear.

Reinforcement learning is used to learn the optimal combination of PTO damping and stiffness coefficients in each sea state without considering any internal models of the device dynamics. The particular variant of reinforcement learning utilized here is called Q-learning. In Q-learning, by agent and environment interaction, the controller learns the optimal behavior, or PTO control policy. Because the space and environment are continuous, neural networks are combined with Q-learning to consider all sea states in addition to helping the model converge faster to optimal behavior. Neural networks are the agent that learns to map state-action pairs to rewards. A block diagram of the overall control structure is shown in Figure 3.

At the heart of Q-learning is the Q function, which maps state-action pairs to the highest combination of current and future rewards (in this case, power produced).

$$Q(s_t, a_t) \leftarrow (1 - \alpha_k)Q(s_t, a_t) + \alpha_k\big(R(s_t, a_t) + \gamma Q(s_{t+1}, a_{t+1})\big) \qquad (23)$$

The Q function, which is a function of the current state and action, is updated with some combination of the previous known value of the state and action pair at time t along with the current reward (i.e., the value of the current state and action pair) and a weighted consideration of the value of future rewards.

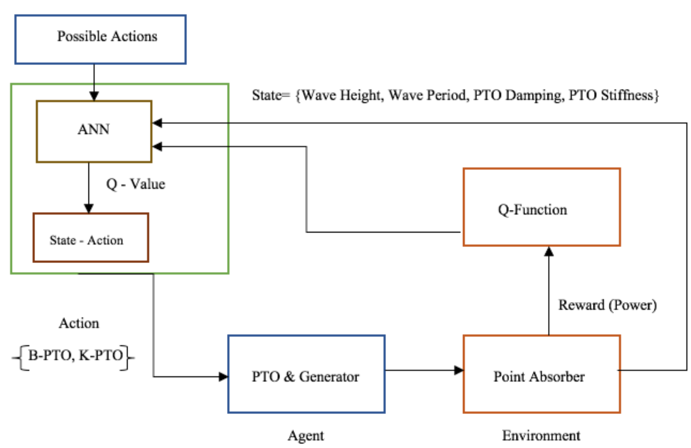

Figure 3: reinforcement learning block diagram.

The parameters alpha and gamma control the weight of current vs. previous information, and the value of future considerations, respectively. The reward is whatever the designer deems desirable behavior, which in a wave energy application will likely be power with some penalty for operating at or beyond system limits.

Figure 4 and Figure 5 below show simulated results of the Q-learning algorithm adaptively finding the optimal PTO damping and stiffness for two cases: regular and irregular waves. It is demonstrated that the algorithm is able to find the optimal stiffness and damping. In actual application, the optimal stiffness and damping for the current sea state would not be known a priori.

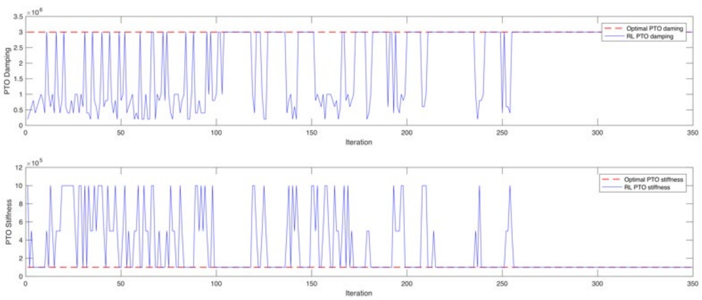

Figure 4: reinforcement learning converging to the optimal PTO stiffness and damping in regular seas.

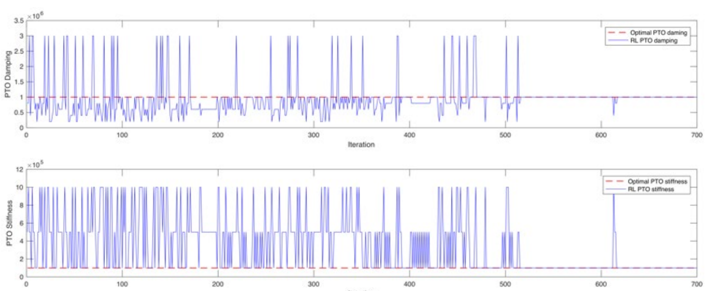

Figure 5: reinforcement learning converging to the optimal PTO stiffness and damping in irregular seas conditions.

5.2 Fuzzy Logic

Fuzzy logic is a modeling and control approach in which the system state and actions are classified as belonging to sets and following logical rules based on plain language heuristics and expert knowledge. For example, wave energy converter speed, which may range from -1 m/s to 1 m/s in normal operation, could be classified such that speeds between 1 m/s and 0.25 m/s are "large positive", speeds between 0.75 m/s and 0 m/s are classified as "small positive", and so on in a similar fashion for the negative speeds. It is noted that in this example, the boundaries of the sets overlap. In fuzzy logic, partial set membership is allowed. For example, a speed of 0.5 m/s could be classified as being 33% membership of the "large positive" set and 66% membership of the "small positive" set.

The control portion of fuzzy logic is the resolution of logical statements that dictate actions to be taken. For example, for something like simple linear damping, the control laws could be: if the speed is large positive, the PTO force is large negative (assuming PTO force is defined as positive in the same direction as motion), and if the speed is small positive, the PTO force is small positive. If the speed was 0.5 m/s, as given in the example above, we can see that we need to apply 33% of the first rule and 66% of the second rule. There are several different methods to resolve this, the simplest of which is to simply apply 33% of whatever is defined as a large negative PTO force and 66% of whatever is defined as a small negative PTO force.

The chief advantages of fuzzy logic are that it can handle highly non-linear control that would otherwise be difficult to specify as a continuous function, and it is relatively easy to specify and deploy in applications in which a human expert operator has a relatively good understanding of stable and desirable behavior.

Using the existing RM3 model in WEC-Sim, a fuzzy control system implementing over-travel protection on top of a traditional linear PTO damping strategy was developed. The Fuzzy Inference System was built using MATLAB's Fuzzy Logic Toolbox and supplied the damping coefficient for the force-actuated linear-PTO in WEC-Sim. Figure 6 shows the input membership functions used for speed and position. The operating ranges for these inputs are selected based on the performance of a linearly damped point absorber under nominal wave conditions, which were assumed as a 2 meter wave height and a period of 10 seconds.

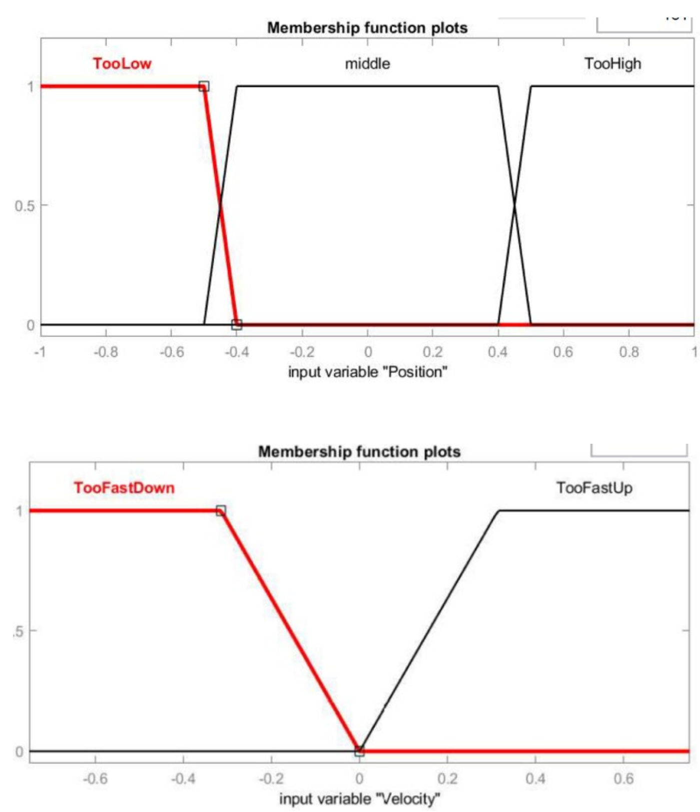

Figure 6: example position and velocity fuzzy membership functions.

The relative velocity of the float is considered "too fast'" when it reaches the maximum speed achieved at the expected wave conditions, as such both membership functions, the positive and negative directions, become fully true at this time.

The position membership functions include the “middle,” considered anywhere in the expected operating range, and upper and lower over-travel limits, which become fully true at the edges of the nominal range. When the float begins to travel outside the nominal range, the controller should respond by over-damping the float to keep it within the limits. The following rules will achieve this

- If position is middle, then PTO force is linearly damped
- If position is too high and velocity is too fast up, then PTO force is very over-damped
- If position is too low and velocity is too fast down, then PTO force is very over-damped

Figure 7 shows the float's relative position under the nominal sea state, for a linearly damped model, and a linear damped model including the fuzzy over-travel protection. The result of the over-damping for position control is a compromise of 7.31% of the captured energy. It may be possible to decrease the losses incurred while in the nominal range through tuning, with the trade-off of subjecting the PTO to more sudden forces. A similar plot is shown for a wave height of 3 meters in Figure 8. This plot shows that the fuzzy logic controller is capable of limiting the motion of the float in high wave heights, while maintaining comparable power performance to a linearly damped PTO in the operating nominal range.

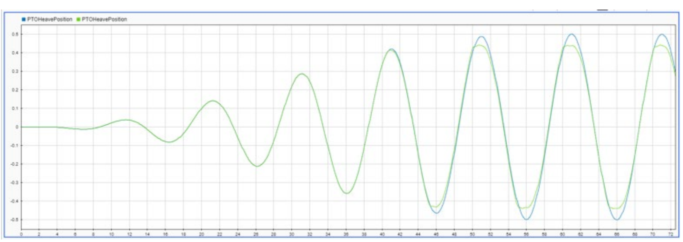

Figure 7: float position under linearly damped control, and fuzzy control to limit overtravel for system protection, with 2 meter regular seas.

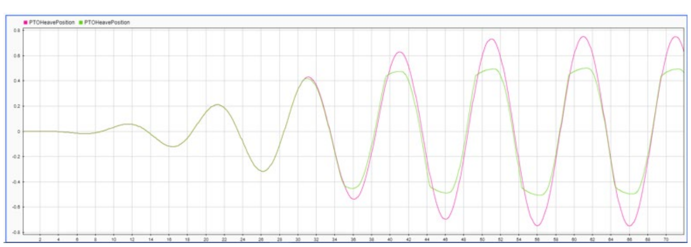

Figure 8: float position under linearly damped control, and fuzzy control to limit overtravel for system protection, with 3 meter regular seas.

6 WAVE BASIN TEST DATA FOR ANALYTICAL AND NUMERICAL MODEL VALIDATION

In this section, existing undisturbed wave test data available for use in this project study are described, as well as the identification of additional undisturbed wave tests necessary to validate the nonlinear models presented in the previous sections. The identification of suitable undisturbed wave tests also considers the availability of experimental data that includes a physical model device where the wave-structure interaction has been measured (wave excitation, scattering and radiation), the device dynamic properties has been characterized (stiffness, damping and added mass), and the Wave Energy Converter includes measurements of a Power Take-Off. In this way, not only the analytical and numerical models of the wave propagation can be validated, but also the simulation models of the device dynamics and PTO performance, both to be implemented in WEC-Sim.

Finally, the identification of existing test data for model validation will provide a basis for additional experiments to be conducted to extend the database, particularly with the vision of aiming nonlinear conditions of waves, PTO control strategies, and mooring system response.

6.1 Existing Wave, WEC and PTO Control Test Data

To perform the analytical and numerical model validation, only experimental data available from previous projects executed at the O.H. Hinsdale Wave Research Laboratory (HWRL) were considered. A significant amount of experiments involving water wave propagation have been carried out in the HWRL over the last years, where detailed data has been collected in a broad range of applications, including studies of Wave Energy Converters, floating structures, wave-structure interaction, and wave propagation and hydrodynamics. In principle, all of them are suitable for model comparison. However, wave generation techniques, availability of data, existence of cases with and without the model, and tests related to Marine Energy and measurements of PTO performance, have reduced the selection of those cases presented herein.

Firstly, only studies carried out after 2015 have been considered. The reason behind is the significant improvement of wave generation techniques implemented in the laboratory at this time. Nonlinear regular waves have been simulated following [29] and [30], and second-order compensation has also been programmed for regular and irregular wave generation according to [31] and [32].

Wave Energy Devices have been tested in both facilities (i.e. the Large Wave Flume –LWF- and the Directional Wave Basin –DWB-) at HWRL. In general, preference in the selection has been given to experiments performed at the DWB given the three-dimensional character of studies associated to marine energy. These cases are suitable for testing nonlinear wave generation techniques, nonlinear wave propagation, nonlinear PTO response, and nonlinear mooring system analysis.

Depending on the project objectives and scope, the experiments selected include tests with and without the presence of a physical model, specimen or device.

Finally, identification of test cases with measurements of a PTO is also considered relevant for further model comparisons.

In Table 1, selected projects suitable for the analytical and numerical model validation, and executed at the HWRL since 2015, have been enlisted where the main selection characteristics are included. Interestingly, the wave conditions tested for the

WEC-Sim validation tests [33] is a super-set of the ALFA OWC [34] experiments. NOWSim were a series of tests performed during the second phase of ALFA OWC, where nonlinear waves and breathers were tested as part of the development of the present study.

Figure 9 present the different regular wave cases available. The wave conditions are presented in dimensionless form and compared with the regions of validity of the different analytical wave theories. As seen in Figure 9, the available test data considers deep and intermediate water depths, with some weak nonlinear cases.

Table 1: Selected experimental data for model validation.

Project	Regular waves	Irregular waves	Undisturbed conditions	PTO measurements
WEC-Sim [33]	●	●	●	●
ALFA OWC [34]	●	●		●
ALFA OWC 2	●	●	●	●
NOWSim		●	●	

Finally, Figure 10 presents the available irregular wave cases for numerical model validation. The individual waves of the time series have been obtained by means of a standard zero-crossing analysis and the significant wave height and energy period is also indicated for comparison purposes.

As seen, an irregular case includes a broad range of individual waves with varying nonlinearity. Here is where the assumptions in the model of propagation of waves by means of a linear superposition of harmonics may fail significantly, since the nonlinear interaction of the different components is not taken into account.

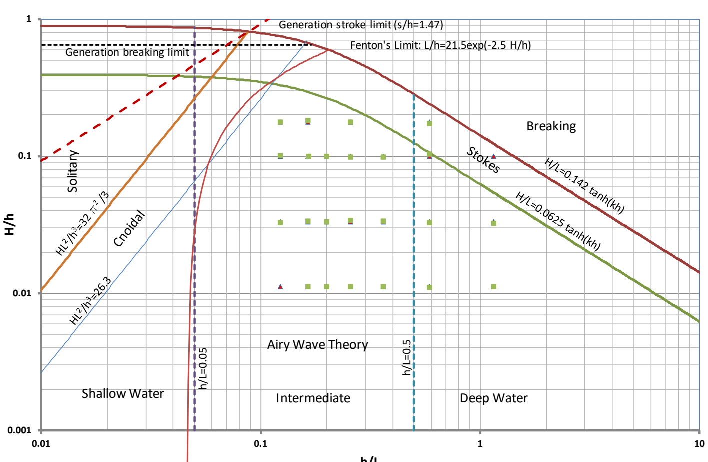

Figure 9: Existing regular wave data sets for analytical and numerical model validation.

6.2 Additional Wave, WEC and PTO Control Tests

As indicated previously, the development of the different nonlinear models for wave propagation, PTO control and mooring system design presented herein, require specific physical model experimentation for validation purposes. The existing data sets selected consider primarily linear or weakly nonlinear wave conditions, the PTO control strategy implemented was oversimplified, and no mooring system was included. Moreover, new advances in nonlinear wave generation in the laboratory have not been implemented yet.

Hence, additional wave, WEC, PTO control and mooring system tests under nonlinear conditions are necessary to fully validate the proposed analytical and numerical models.

The additional tests will be executed in the Directional Wave Basin at the HWRL, will include undisturbed nonlinear wave conditions with fully nonlinear wave generation, as well as testing with a complex WEC model that includes nonlinear PTO control strategies, and a catenary mooring system.

These tests will extend the existing database for WEC-Sim validation, and will be designed to validate independently the different models proposed.

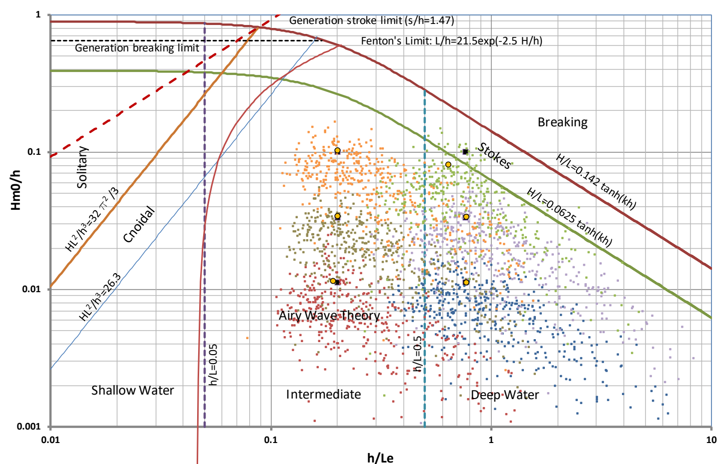

Figure 10: Existing irregular wave data sets for analytical and numerical model validation.

Summarizing, the available data set for analytical and numerical validation includes 23 different regular wave conditions (H=0.015 m to 0.242 m, T=0.87 s to 3.307 s), and 6 irregular wave conditions (H_{m0}=0.015 m to 0.136 m, T_p=1.219 s and 2.611 s), at a single water depth of 1.36 m.

7 FIELD MEASUREMENT DATA FOR ANALYTICAL AND NUMERICAL MODEL VALIDATION

Processing of the field data has already shown the importance of nonlinearity in the wave field. Figure 11 shows that observed wave elevations have more extremes (and thus more kurtosis) than linear reconstructions of the observed conditions. Processing of the field data has also identified breaking waves and quantified the exceedance of these motions relative to linear theory [35]. Future work will use these data products to validate the nonlinear wave maker theory results and ensure that realistic time series are generated for laboratory experiments.

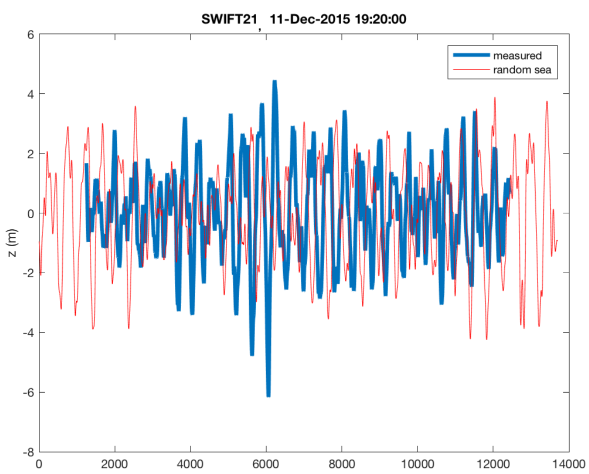

Figure 11: Measure (blue) and simulated (red) time series of sea surface elevations during a large wave event off the Oregon Coast in December 2015. The measured time series is from a SWIFT buoy and includes a rogue wave at index 6000, which is absent from the simulated time series created from a linear model with the same measured spectrum. Samples are at 25 Hz.

8 CONCLUDING REMARKS

The overall research plan and preliminary results obtained to date from the nonlinear effects on WEC dynamics project have been summarized in this study. The resulting models, algorithms and solution procedures will be applied to an experimental WEC system well-familiar to our group and the WEC research community. Existing experiments from the selected system will be used to validate the models. In particular, we described the model development and preliminary progress of the on-going research study on the effects of nonlinearities in ocean wave input and power-take-off (PTO) control on wave energy conversion system dynamics and efficiency. The model system employed and progress on recent developments were: (1) nonlinear wave modeling in the ocean, generation and propagation in a wave basin, and (2) nonlinear PTO control algorithm. An overview of the holistic analytical, numerical and experimental research approach is presented. To provide a simple means for analysis, comparison and performance evaluation, the WEC-Sim numerical platform was used for model implementation and system dynamic simulation. Analytical and numerical predictions of the nonlinear wave fields in a wave basin using the nonlinear Fourier analysis (NLFA) technique and corresponding nonlinear wavemaker theory will be validated using a comprehensive series of experimental test data as well as ocean wave measurements. Efficiency of the nonlinear PTO control will be evaluated by comparing numerical simulations with results of WEC model test data under corresponding wave conditions of the experimental studies without the presence of the WEC system. Additional experiments to complement the existing ones will be conducted in the coming year to validate the nonlinear wave models and PTO algorithms. Results from these planned studies, together with the improvements on nonlinear mooring line dynamic analysis procedures, will be reported in future conferences.

ACKNOWLEDGEMENTS

The authors are grateful for the financial support from the Department of Energy ALFA-LCO Grant No.DE-EE0006816.

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