Resonances in an Evolving Hole in the Swash Zone

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Abstract: Water oscillations observed in a 10-m-diameter, 2-m-deep hole excavated on the foreshore just above the low-tide line on an ocean beach were consistent with theory. When swashes first filled the initially circular hole on the rising tide, the dominant mode observed in the cross-shore velocity was consistent with a zero-order Bessel function solution (sloshing back and forth). As the tide rose and swash transported sediment, the hole's diameter decreased, the water depth inside the hole remained approximately constant, and the frequency of the sloshing mode increased according to theory. About 1 h after the swashes first reached the hole it had evolved from a closed circle to a semicircle, open to the ocean. When the hole was nearly semicircular, the observed cross-shore velocity had two spectral peaks, one associated with the sloshing of a closed circle, the other associated with a quarter-wavelength mode in an open semicircle, both consistent with theory. As the hole evolved further toward a fully semicircular shape, the circular sloshing mode decreased, while the quarter-wavelength mode became dominant. **DOI: 10.1061/(ASCE)WW.1943-5460.0000136.**

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Introduction

Enclosed and semienclosed bodies of water resonate at the frequencies of their natural modes. The resonant frequencies depend on the basin geometry (Strutt 1877; Lamb 1916; Rabinovich 2009, and references therein), and have been investigated in estuaries, harbors, inlets, fjords, and bays (Wiegel 1964; Raichlen 1966; Miles 1974; Raichlen and Lee 1992; Okihiro and Guza 1996, and references therein, among others). The response of harbors to a range of forcing frequencies and the change in resonance owing to various entrance channel configurations have been investigated theoretically (Miles and Munk 1961; Unluata and Mei 1975; Wu and Liu 1990, and many others). Small, artificial structures designed to resonate at specific frequencies also have been investigated theoretically (Valembois 1953; Lates 1963; James 1970a), as well as with laboratory experiments (Valembois 1953; James 1970b, c, 1971a, b). Resonators have been constructed along entrance channels to protect harbors from seiches driven by waves from the open ocean. Field studies of these structures have focused on the sediment transported by the currents associated with the resonant mode (Donnelly and MacInnis 1968; Prandle 1974). However, there have been few, if any, field observations of resonances in small artificial structures forced by broadband ocean wind waves. Here, it is shown that as the shape of a hole excavated near the low-tide line on an ocean beach evolves from a closed circle to a semicircle open to the ocean, the water oscillates at the theoretically predicted natural modes of the system.

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Field Observations

A 10-m-diameter, 2-m-deep circular hole [Figs. 1(a) and 2(a)] was excavated with a backhoe slightly above the low-tide line on a relatively long, straight Atlantic Ocean beach on the Outer Banks of North Carolina, near Duck. About 1 h after the excavation was completed, the tide had risen sufficiently for swash running up the beach to reach the sides of the hole and fill it with water [Fig. 1(a)]. As the tide rose further, more swash reached the hole, which developed a small gap on the seaward edge [Fig. 1(b)]. As the swash transported sediment, the hole filled primarily from the sides, its wetted diameter decreased [Fig. 2(c)], the gap enlarged [Fig. 1(c)], and the closed circle evolved toward an open semicircle [Figs. 1(d) and 2(b)]. The water depth inside the hole remained approximately constant [Fig. 2(c)] because the tide was rising as the hole filled with sediment. About 2 h after the swash first reached the hole, it was refilled with sediment, and the bathymetry of the hole location and the surrounding beach was similar to the bathymetry observed before the excavation.

The hole and surrounding beach were surveyed continually with a global positioning system carried by walkers (horizontal accuracy of ± 0.10 m; vertical accuracy of ± 0.03 m), and every 12 min the sand levels at the seven sensor locations in the hole [Figs. 1 and 2(a)] were measured by hand (accuracy ± 0.02 m), and the instruments were raised to prevent burial as the hole filled. The initial volume of the excavated hole was approximately 157 m^3. The removed sand was stored above the high-tide line, where it remained during the experiment. Beach surveys did not indicate changes in sediment levels alongshore of the excavation site, suggesting the sediment that refilled the hole was transported primarily from offshore. The hole refilled with sand in less than 2 h, implying a total infilling rate of approximately 0.02 m^3/s and a transport rate across the 10-m diameter of roughly 0.02 m^3/s per m. Consistent with previous swash zone studies (Watts 1953; Beach and Sternberg 1991), these sediment transport rates were an order of magnitude greater than those observed in the inner and outer surf zones during

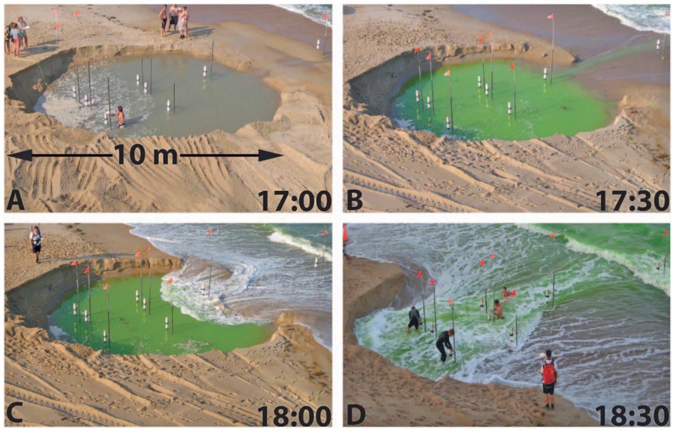

Fig. 1. (Color) Photographs of the excavated hole at different times [hours (EST) on September 19, 2005, are listed]: the white cylinders on the vertical pipes are current meters

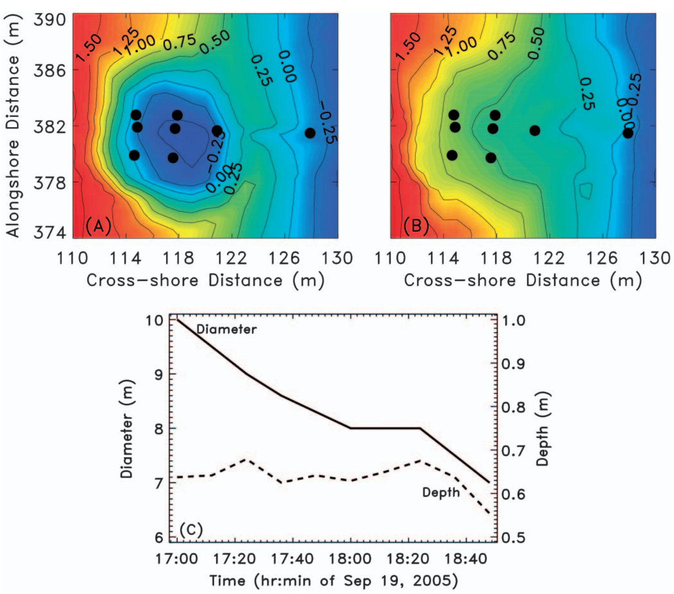

Fig. 2. (Color) Bathymetry (colors and contours) of the hole at approximately (a) 1700 hrs. and (b) 1830 hrs. as a function of cross- and alongshore distances; labeled contour lines are drawn every 0.25 m from about 1.75 (red) to −0.25 m (blue) relative to mean sea level; symbols are the locations of the current meters; (c) diameter of the wetted perimeter (solid curve, left axis) and water depth inside the hole (dashed curve, right axis) versus time

strong storms on this beach (Thornton et al. 1996; Gallagher et al. 1998).

An array of pressure gauges in 8-m depth, 800 m offshore of the shoreline was used to estimate the incident waves. Seven acoustic Doppler current meters were deployed in the hole (Figs. 1 and 2) and three additional current meters with colocated pressure sensors (not shown) were deployed at locations extending from the hole to a 0.8-m-high sandbar in 1.2-m water depth. Incident-significant wave heights in 8-m depth were about 0.4 m, with most of the energy in a narrow swell peak with frequency $f = 0.1$ Hz, and approached the beach about 28° from normal incidence. The incident waves did not change significantly during the 2-h time period studied here. Refraction and shoaling resulted in the 0.4–0.5-m high waves propagating within 5° of normal incidence that were observed in 1.2-m depth, 80 m offshore of the hole. Cross- and alongshore currents were less than 0.15 m/s in 1.2-m depth, and less than 0.10 m/s in the hole, except when the hole had evolved to a semicircle, at which time cross-shore currents near the seaward edge increased to 0.50 m/s. Visual observations (the green water in Fig. 1 is dye) suggest the strong offshore-directed cross-shore flows near the open edge of the semicircle at least partially were owing to a circulation pattern similar to that observed in beach cusps (Lane 1888; Evans 1938, 1945; Bowen and Inman 1971; Guza and Inman 1975), with the swash running up the beach on both sides of the hole being directed into the hole, and out along the center [Fig. 1(d)] showing strong seaward directed outflow.

Energy density spectra from the 512-s long time series of cross-shore velocity observed in the hole have peaks that do not always coincide with the frequency of the incident wind waves (Fig. 3). Specifically, although between 1700 and 1900 hrs. the incident waves (observed in 1.2 m depth, 80 m offshore of the hole) were dominated by $f = 0.10$ Hz swell (black curve, Fig. 3), the motions inside the hole had spectral peaks that ranged from near $f = 0.14$ Hz (red curve, Fig. 3) when water first entered the hole [1700 hrs., Fig. 1(a)] to $f = 0.16$ Hz (green curve, Fig. 3) to $f = 0.18$ Hz (blue curve, Fig. 3), to $f = 0.10$ (pink curve, Fig. 3) as the hole evolved (Fig. 1). Although the confidence limits on the spectral estimates with 20 degrees of freedom (obtained by merging 10 neighboring frequencies in periodogram estimates of the spectra, resulting in a frequency resolution of approximately 0.02 Hz) were large (90% bars in Fig. 3), the peaks are statistically significant. Alongshore velocities have spectral peaks at frequencies similar to those for cross-shore velocities, as well as at other frequencies. However, the alongshore velocities were about an order of magnitude smaller than the cross-shore velocities, resulting in less confidence in the spectral estimates and, thus, are not considered further.

Theory

The water surface in a circular basin has many resonant modes (Lamb 1916), similar to acoustic resonators (Strutt 1877). The lowest mode is when the water sloshes back and forth from one side of

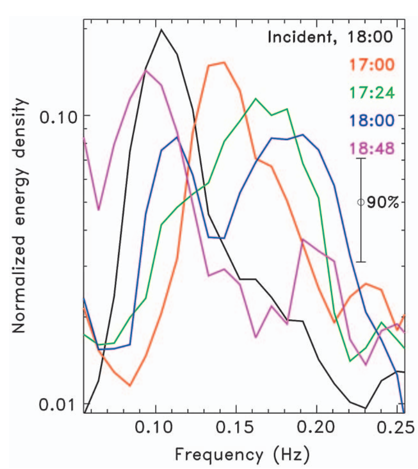

Fig. 3. (Color) Energy density (normalized by the total variance) of cross-shore velocity observed near the center of the hole (colored curves are different times) and 80 m offshore (black curve) versus frequency [the times (hours (EST) on September 19, 2005, are listed]

the circle to the other (i.e., a half-wavelength mode). The frequency of this mode is given by (Lamb 1916, Article 191)

$$f = \frac{\sqrt{gh}}{3.4r} \tag{1}$$

where g = gravitational acceleration, h = water depth, r = radius of the circle, and the constant 3.4 comes from the roots of the first-order asymmetrical Bessel function solutions for a sloshing mode. Changes in surface elevation are minimum (node) and horizontal velocities are maximum (antinode) along an alongshore line through the center of the hole (i.e., the sloshing is in the cross-shore direction).

Unlike closed basins, in which the lowest mode corresponds to a half-wavelength, the lowest mode in a basin open to the ocean is a quarter-wavelength resonance (Korgen 1995) in which there is an antinode in the surface elevation (a node in the cross-shore velocity) at the closed edge, and a node in the surface elevation (an antinode in the cross-shore velocity) at the open boundary. The frequency of the quarter-wavelength resonator in a semicircular basin open to the ocean is given by (Wilson 1972; Rabinovich 2009)

$$f = \frac{\sqrt{gh}}{4.44L} \tag{2}$$

where L = radius (the distance from the open boundary to farthest point of the semicircle) and the constant 4.44 comes from an approximate zero of the lowest-order Bessel function solution for a semicircular basin. If the open basin is rectangular, the constant is 4.0.

Results

The observed spectral peaks in the cross-shore velocity were consistent with theoretical predictions (Fig. 4). When the hole was a closed circle [Figs. 1(a) and 2(a)], the water surface oscillated at the predicted [Eq. (1)] frequency ($f = 0.14$ Hz) of the lowest mode (sloshing) for the observed geometry and water depth [Fig. 2(c)], forced by occasional swash overwashing the sides

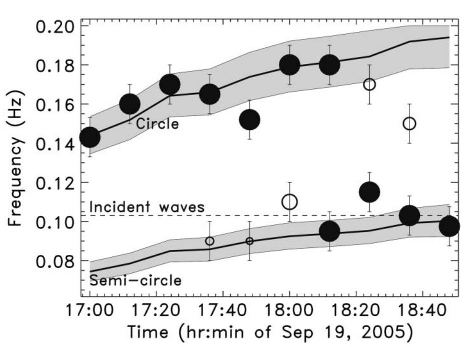

Fig. 4. Frequency of observed (symbols) and predicted (solid curves) resonances in the cross-shore velocity for a closed circular basin and an open semicircular bay with the observed diameters and depths versus time; solid circles indicate the frequency of the largest (or the only) spectral peak for that time period, whereas open circles indicate the frequency of a second, smaller spectral peak, with the diameter of the open circle proportional to the area under the smaller peak relative to that under the larger peak; gray shaded areas encompass predictions for ±0.25 m uncertainties in diameter and ±0.05 m uncertainties in depth

(see Fig. 4, where the upper solid curve is the predicted frequency for the lowest-order resonant mode for a closed circle given the observed changing diameter and water depth). The phase relationships between the spatially separated (the sensors in Figs. 1 and 2) observations of the cross-shore velocity at the resonant peak were consistent with a lowest-order mode, with water sloshing back and forth in the cross-shore direction. As the tide rose and the sediment was transported by swash, the diameter of the wetted circle decreased [Fig. 2(c)], the water depth inside the hole changed only slightly [Fig. 2(c)], and the observed and predicted resonant frequencies increased (Figs. 3 and 4, time = 1700–1824 hrs.).

When the hole was primarily a closed circle (1700 < time < 1800 hrs., Figs. 1 and 2), there was one spectral peak (Figs. 3 and 4). As the hole shape evolved toward a semicircle, the observations were consistent with an increase in the relative amplitude of the quarter-wavelength mode for an open semicircle (see Fig. 4, where the lower solid curve is the predicted frequency for the lowest-order resonant mode for an open semicircle given the observed changing diameter and water depth), and with a decrease in the relative amplitude of the sloshing mode for a closed circle (Fig. 4). When the hole is a semicircle [Figs. 1(d) and 2(b)], the frequency of the observed spectral peak is consistent with the frequency of the theoretical quarter-wavelength mode (Fig. 4, time = 1848 hrs.).

The observed spectral peaks do not all fall precisely on the theory curves (Fig. 4), partially owing to errors associated with short data records with finite bandwidth spectral estimates [the bars on the symbols in Fig. 4 are the width of the frequency band (approximately 0.02 Hz) for spectral estimates of 512-s long records with 20 degrees of freedom]. In addition, during the 512-s data records, the hole evolved, with the diameter and depth changing, resulting in a broadening of the resonant frequency peak. However, most of the observed spectral peaks have frequencies that fall within the range of predicted frequencies for ±0.25 m uncertainties in diameter (r) or length (L) and ±0.05 m uncertainties in depth (h) (the gray shaded areas surrounding each solid theory curve in Fig. 4).

Although the observed spectral peaks for the open semicircle were consistent with theory ($f \approx 0.09$ Hz, Fig. 4), they also were close to the frequency of the incident waves ($f \approx 0.10$ Hz, Fig. 3 black curve; Fig. 4 dotted line). If the oscillations observed near the open edge of the semicircle were owing to progressive incident waves, there would be a spectral peak in both cross-shore velocity and sea-surface elevation (or, similarly, vertical velocity) fluctuations. In contrast, the observations were consistent with a standing wave at the quarter-wavelength resonance frequency ($f = 0.09$ Hz), with a spectral peak (antinode) in the cross-shore velocity and a spectral trough (node) in the sea-surface elevation and vertical velocity (Fig. 5). The coherence and phase (not shown) between the spatially separated sensors along the cross-shore transect extending 80 m offshore from the hole were consistent with standing waves at frequencies below about $f = 0.06$ Hz (the peaks and troughs in the cross-shore and vertical velocity fluctuations at $f = 0.06$ Hz) (Fig. 5) and progressive waves at higher frequencies (e.g., Fig. 9 in Elgar and Guza 1985). Thus, the observations after about time = 1830 hrs., when the hole was an open semicircle [Figs. 1(d) and 2(b)], were consistent with a quarter-wavelength standing wave with frequency $f = 0.09$ Hz (Figs. 4 and 5), and were not an artifact of swash with the incident wave frequency $f = 0.10$ Hz entering the hole.

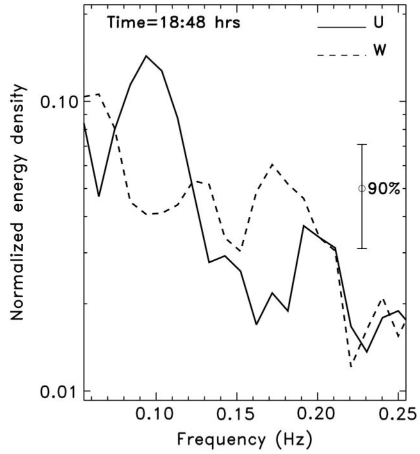

Fig. 5. Energy density (normalized by the total variance) of cross-shore (U) and vertical (W) velocity observed at the open boundary of the semicircular hole versus frequency

Conclusions

The water surface and velocities in a 2-m-deep, 10-m-diameter hole excavated near the low-tide line on an ocean beach oscillated at the frequency ($f = 0.14$ Hz) of a half-wavelength sloshing mode in a circular basin even though the incident waves had frequency $f = 0.10$ Hz. As the hole diameter decreased owing to sediment influx at the hole edges, the cross-shore sloshing increased in frequency (to $f = 0.17$ Hz) according to theory. About 1 h after swash started to impact the hole, it began to evolve to a semicircular shape open on the seaward boundary, and the cross-shore velocities fluctuated at a lower frequency ($f = 0.09$ Hz) associated with a quarter-wavelength resonator, as well as at the closed-circle sloshing frequency. As the semicircle became more open, the half-wavelength mode decreased in strength and disappeared, while the quarter-wavelength mode increased in strength and in frequency, as predicted by theory.

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References

Beach, R., and Sternberg, R. (1991). "Infragravity driven suspended sediment transport in the swash, inner and outer surf zone." *Proc., Coastal Sediments*, ASCE, 114–128.

Bowen, A. J., and Inman, D. L. (1971). "Edge waves and crescentic bars." *J. Geophys. Res.*, 76(36), 8662–8671.

Donnelly, P., and MacInnis, I. (1968). "Experience with self-dredging harbour entrances." *Proc., 11th Int. Conf. on Coastal Engineering*, ASCE, 1283–1294.

Elgar, S., and Guza, R. T. (1985). "Shoaling gravity waves: Comparisons between field observations, linear theory, and a nonlinear model." *J. Fluid Mech.*, 158, 47–70.

Evans, O. F. (1938). "The classification and origin of beach cusps." *J. Geol.*, 46(4), 615–627.

Evans, O. F. (1945). "Further observations on the origin of beach cusps." *J. Geol.*, 53(6), 403–404.

Gallagher, E. L., Elgar, S., and Guza, R. T. (1998). "Observations of sand bar evolution on a natural beach." *J. Geophys. Res.*, 103(C2), 3203–3215.

Guza, R. T., and Inman, D. L. (1975). "Edge waves and beach cusps." *J. Geophys. Res.*, 80(21), 2997–3012.

James, W. (1970a). "Rectangular resonators for harbor entrances." *Proc., 11th Conf. on Coastal Engineering*, ASCE, 1512–1530.

James, W. (1970b). "Resolution of partial clapotis." *J. Wtrwy., Harb. and Coast. Engrg. Div.*, 96(1), 165–170.

James, W. (1970c). "Spectral response of harbor resonator configurations." *Proc., 12th Coastal Engineering Conf.*, ASCE, 2181–2193.

James, W. (1971a). "Two innovations for improving harbor resonators." *J. Wtrwy., Harb. and Coast. Engrg. Div.*, 97(1), 115–122.

James, W. (1971b). "Response of rectangular resonators to ocean wave spectra." *Proc. Inst. Civ. Eng.*, 48(1), 51–63.

Korgen, B. (1995). "Seiches." *Am. Sci.*, 83, 330–341.

Lamb, H. (1916). *Hydrodynamics*, 4th Ed., Cambridge University Press, Cambridge, U.K.

Lane, A. (1888). *The geology of Nahant*, Boston Society of Natural History, Boston, 91–95.

Lates, M. (1963). *Recherches hydrauliques de laboratoire sur l'efficacite de quelques types d'ouvrages de protection des petits ports maratimesc contre la penetration des vagues et des alluvions chariees*, L'Institut d'etudes et de Recherches Hydrotechniques, Bucharest, Romania (in French).

Miles, J. W. (1974). "Harbor seiching." *Annu. Rev. Fluid Mech.*, 6, 17–33.

Miles, J. W., and Munk, W. H. (1961). "Harbor paradox." *J. Wtrwy. and Harb. Div.*, 87(3), 111–132.

Okihiro, M., and Guza, R. T. (1996). "Observations of seiche forcing and amplification in three small harbors." *J. Waterway, Port, Coastal, Ocean Eng.*, 122(5), 232–238.

Prandle, D. (1974). "Wave resonators for harbour protection." *Dock Harbour Auth.*, 55(650), 279–280.

Rabinovich, A. B. (2009). "Seiches and harbor oscillations." *Handbook of coastal and ocean engineering*, Y. C. Kim, ed., World Scientific, Singapore, 193–236.

Raichlen, F. (1966). "Harbor resonance." *Estuary and coastline hydrodynamics*, A. T. Ippen, ed., McGraw-Hill, New York, 281–340.

Raichlen, F., and Lee, J. J. (1992). "Oscillation of bays, harbors and lakes." *Handbook of coastal and ocean engineering*, J. B. Herbich, ed., Gulf Publishing, Houston, 1073–1113.

Strutt, J. W. (1877). *The theory of sound*, 3rd Baron Rayleigh, Macmillan, London.

Thornton, E. B., Humiston, R. T., and Birkemeier, W. (1996). "Bar/trough generation on a natural beach." *J. Geophys. Res.*, 101(C5), 12097–12110.

Unluata, U., and Mei, C. C. (1975). "Effect of entrance loss on harbor oscillations." *J. Wtrwy., Harb. and Coast. Engrg. Div.*, 101(2), 161–180.

Valembois, J. (1953). "Etude de l'action d'ouvrages resonants sur la propagation de la houle." *Proc., Minnesota Int. Hydraulics Conf.*, Minneapolis, 193–200.

Watts, G. M. (1953). "Field investigation of suspended sediment in the surf zone." *Proc., 4th Conf. on Coastal Engineering*, ASCE, 181–199.

Wilson, B. W. (1972). "Seiches." *Advances in hydroscience*, Vol. 8, Academic, New York, 1–94.

Wiegel, R. L. (1964). "Tsunamis, storm surges, and harbor oscillations." Chapter 5, *Oceanographical engineering*, Prentice-Hall, Englewood Cliffs, NJ, 95–127.

Wu, J.-K., and Liu, P. L.-F. (1990). "Harbor excitations by incident wave groups." *J. Fluid Mech.*, 217, 595–613.