Tunable evolutions of wave modes and bandgaps in quasi-1D cylindrical phononic crystals

Mehrashk Meidani a, Eunho Kim b, Feng Li, Jinkyu Yang b,*, Duc Ngo c

a Department of Civil & Environmental Engineering, University of Massachusetts, Amherst, MA 01003, USA
b William E. Boeing Department of Aeronautics & Astronautics, University of Washington, Seattle, WA 98195, USA
c School of Engineering, Eastern International University, Binh Duong, Vietnam

ABSTRACT

We investigate the tunable characteristics of mechanical waves propagating in quasi-1D phononic crystals composed of horizontally stacked short cylinders at various contact angles and offsets. According to the Hertzian contact theory, elastic compression of laterally-touching cylindrical bodies exhibits a various range of contact stiffness depending on their alignment angles. In this study, we first assemble cylindrical particles in various combinations of inclination angles and systematically examine their forming mechanisms of frequency bandgaps. We also investigate the effect of the rattling motions of cylindrical particles by introducing asymmetric center-of-mass offsets with respect to their contact points. We find that the frequency responses of these quasi-1D phononic crystals evolve into multiple band structures as we employ higher deviations of contact angles and offsets. We calculate the dispersive behavior of propagating waves using a discrete particle model for simple zero-offset cases, while we use a finite element method for simulating the rattling motions of particles under non-zero offsets. We report branching behavior of frequency band structures and the evolution of their vibration modes as we manipulate the contact angles and offsets of the phononic crystals. This study implies that we can leverage the versatile wave filtering characteristics of quasi-1D phononic crystals to construct tunable wave filtering devices for engineering applications.

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1. Introduction

Phononic crystals are defined as periodic architectures of tightly packed particles/rods/slabs, which can manipulate the propagation of mechanical vibrations called phonons [1,2]. They have gained significant attention from the scientific and engineering communities in the last decades owing to their capability of controlling mechanical waves [3–9]. More specifically, these lattice structures forbid the propagation of mechanical waves at certain frequency ranges due to their destructive interferences at interfaces, while allowing the passage of the rest of frequency contents. We refer to these forbidden bands as stop bands or bandgaps, and the allowable frequency bands are called pass bands. From a physical viewpoint, these band structures in phononic crystals are analogous to the optical bandgaps in photonic crystals (e.g., gem stones) and electrical bandgaps in semi-conductors [10].
The filtering properties of phononic crystals, including the width and number of bandgaps, can be tuned by changing their parameters such as material properties, boundary conditions, and geometry [2,6]. In particular, when the mechanical interactions of unit cells are governed by nonlinear contact relationship (e.g., Hertzian contact among spheres), the interfacial stiffness of the lattice structures can be varied by adjusting applied static pre-compression, thereby shifting the cutoff frequencies of band structures in-situ [6,11]. Such tunable nature has also been explored in quasi-1D phononic crystals with glued or welded interfaces [12,13].

While most previous studies have focused on enhancing the tunability of phononic crystals by manipulating lattices’ translational motions, the authors’ recent studies have demonstrated the drastic enhancement of phononic crystals’ tunability by facilitating their rotational dynamics [14]. We observed the emergence of additional pass bands in a mono-atomic chain of short cylindrical particles by triggering rocking motions in addition to the translational motions. This is in contrast to the single pass-band formation found in conventional, mono-dispersed phononic crystals. The versatile variations of frequency band structures in these quasi-1D phononic crystals have also been reported with respect to the alignment angle variations between cylindrical unit cells. Though these findings suggested an additional, useful parameter to manipulate the responses of phononic crystals, the systematic studies on the versatile dynamics of these quasi-1D phononic crystals are demanded.

In this paper, we examine the filtering properties of quasi-1D phononic crystals composed of short cylindrical particles by varying two geometrical parameters: (1) contact angles between cylindrical particles and (2) offsets between the contact point and center-of-mass of individual particles. The first parameter essentially manipulates the contact stiffness between two slanted cylindrical particles. We alternate contact angles in the chain and demonstrate experimentally the formation of additional pass-bands induced by such stiffness variations. This mechanism is conceptually similar to polarizers in optics, which transmit a selected range of electromagnetic waves by rotating their transverse filters. We analyze the wave filtering behavior of phononic crystals and calculate their dispersive mechanisms by using a discrete particle model with state-space approach.

While the second parameter, geometrical offset of the particles, does not affect the contact stiffness, it gives rise to the rotational motion of the cylindrical particles and in turn produces complex, multiple vibration modes in the cylindrical chain. In this study, we elucidate that these branching mechanisms result from the coupling dynamics of cylinders’ translational, rotational and shear motions. We analyze the characteristics of these generated modes and investigate the evolution of such modes, which can be converted from ineffective modes into visible, effective modes and vice versa. To investigate the offset effect, we conduct rigorous finite element simulations, showing how rocking vibrational modes of quasi-1D phononic crystals can induce supplementary pass-band branches and bandgaps. The findings in this study can form a foundation in constructing tunable wave filtering devices based on quasi-1D phononic crystals, which show versatile wave filtering performance for engineering applications.

The rest of the manuscript is structured as follows: In Section 2, we describe the detailed experimental setup. Section 3 explains our numerical approaches based on two different methods: a discrete particle model for calculating wave dispersions in simple, zero-offset chains and a rigorous finite element method for simulating particles’ rattling motions under non-zero offsets. In Section 4, we compare the results of numerical simulations and experimental demonstrations. Lastly, in Section 5, we conclude the manuscript with brief summary and suggested future work.

2. Experimental setup

Fig. 1 shows the schematic and the digital image of the experimental setup composed of a quasi-1D phononic crystal, support rails, and an actuation and sensing system. The phononic crystal consists of 16 cylindrical particles (radius \( R = 9.0 \) mm, height \( H = 18.0 \) mm) made of fused quartz with density \( \rho = 2,187 \) kg/m\(^3\), Young’s modulus \( E = 72 \) GPa, and Poisson’s ratio \( \nu = 0.17 \) [14]. In this study we select fused quartz as the phononic crystal material due to its smooth surface finishing and small energy dissipation compared to other polymeric or metallic materials. The chain of quartz particles is placed on two parallel arrays of slanted L-shaped heavy mass ladders (Fig. 1b). In order to minimize the friction between the cylindrical particles and the ladders, we cover the toes of the ladders with a 0.127 mm-thick, low-friction Teflon tape.

To excite the phononic crystal, we position a piezoelectric actuator (Piezomechanik, model PST 150/14/40 V520) in direct contact with the first particle in the chain. The piezoelectric actuator is firmly fixed to a steel rigid wall and is driven by on-board function generator (NI, model PCI-6112) powered by an external amplifier (Piezomechanik, model LE 150/100 EBW). We use broadband, white noise signals (1.5 s duration) as an excitation waveform. The mechanical waves transmitted through the phononic crystal are measured by a piezoelectric force sensor (PCB, model 208-C01), which is placed on the other end of the chain. We mount the force sensor to a heavy-mass block, which is placed on two stainless steel rollers. This movable wall is in turn compressed by two linear springs (\( k = 1.25 \) kN/m) whose positions are controlled by a linear stage with a 0.01 mm precision. In this study, pre-compression is set to 25.0 N. The position of the actuator and the sensor are adjusted in both horizontal and vertical directions so that their centroids lie on the same line with the particles contact points. The acquired signals from the piezoelectric sensor are first received by the charge amplifier, and its output is recorded by the on-board data acquisition system. We repeat the data acquisition five times and average the measured signals in the frequency domain to enhance their signal-to-noise ratio. Fig. 1a also shows the process of data processing.

To investigate versatile mechanisms of wave transmission through the quasi-1D cylindrical phononic crystal, we systematically vary two geometric parameters: (1) the contact angles between particles, and (2) the offsets of particles’
center-of-mass with respect to their contact points. We begin with manipulating contact angles between the particles. Specifically, we position the cylinders on a pair of parallel holders in a way that they form alternating contact angles of $\alpha_1$ and $\alpha_2$ (Fig. 2a). This is achieved by slanting the angles of two ladders mounted on different angle brackets. In this study, we change $\alpha_1$ from $15^\circ$ to $90^\circ$ in a $15^\circ$ increment, while $\alpha_2$ is kept constant at $90^\circ$. That is, we test six different combinations of alternating contact angles: $[15^\circ, 90^\circ]$, $[30^\circ, 90^\circ]$, $[45^\circ, 90^\circ]$, $[60^\circ, 90^\circ]$, $[75^\circ, 90^\circ]$, and $[90^\circ, 90^\circ]$.

For the second series of tests, the offset of each array of particles, $e_1$ and $e_2$, are varied, while the contact angles between all particles are kept equal to $90^\circ$ (Fig. 2b). We control the offset levels of each array from 0.0 to 7.0 mm in a 0.5 mm step by adjusting the positions of two ladders. We vary their positions in an accurate manner using linear stages attached underneath the ladders (Fig. 1b). It should be noted that the phononic crystal chain with a relatively large offset value tends to be extremely unstable under static pre-compressions of 25 N. If the particles and the sensor/actuator are not perfectly aligned, the chain can easily buckle under dynamic disturbances as small as actuator output. Hence, we mount additional stoppers on the L-shaped rails to hold the top surfaces of the particles and prevent buckling. In normal conditions, the clearance of 0.1 mm is maintained between the top surfaces of the particles and the stoppers. When the phononic crystal is buckled under a large amount of applied offsets, some of the particles may come into contact with these supports. With all these measures, we achieve the maximum offset up to 7.0 mm, which is merely 2.0 mm less than the half-height of the crystal.

Fig. 1. (a) Schematic of the experimental setup composed on a quasi-1D chain of cylindrical particles under pre-compression; (b) Photo of the experimental setup with alternating contact angles ($\alpha_1=30^\circ$, $\alpha_2=90^\circ$) and no offset.
the elliptical contact area, whose ratio can be expressed in terms of the rods’ contact angle, but also governed by their static pre-compression represented by particles) and disturbed by relatively small dynamic excitations (with a dynamic displacement \( \delta \).

The contact stiffness in Eq.(1) is simplified to

\[
\frac{F}{a} = \frac{K(e)}{\pi} \sin \alpha \sqrt{\frac{c}{b}} E(e) - K(e) \left[ K(e) - E(e) \right]^{1/2}
\]

where \( K(e) \) and \( E(e) \) are complete elliptical integrals of the 1st and 2nd types [15]. The parameter \( e \) is the deviation of the contact area shape from a perfect circle, which can be calculated by \( e \approx \pi (1 - \cos \alpha)/(1 + \cos \alpha)^{3/2} \) (e.g., \( e = 1 \) for a circular contact when cylinders are placed at \( 90^\circ \) contact angle). Likewise, \( a \) and \( b \) are the semi-major and -minor axes of the elliptical contact area, whose ratio can be expressed in terms of \( \alpha \) as: \( b/a \approx (1 - \cos \alpha)/(1 + \cos \alpha)^{2/3} \). When \( \alpha = 90^\circ \), the contact stiffness in Eq. (1) is simplified to \( k_{cyl}(90^\circ) = 2E\sqrt{R}/3(1 - \nu^2) \), which is similar to, but larger than the classical Hertzian contact coefficient (i.e., between two identical spheres) by a factor of \( \sqrt{2} \).

If the cylindrical contact is under strong static compression (with a static displacement \( \delta_0 \) between neighboring particles) and disturbed by relatively small dynamic excitations (with a dynamic displacement \( \delta_d \) between adjacent particles), we can linearize the dynamic force–displacement relationship (i.e., \( F_d \) vs. \( \delta_d \)) as:

\[
F_d = \frac{3}{2} k_{cyl}(\alpha) \delta_0^{1/2} \cdot \delta_d
\]

by expanding the nonlinear Hertzian contact relationship up to the first-order term via the Taylor expansion [11]. It should be noted that the slope of this linearized force–displacement curve (i.e., linearized contact stiffness) is not only a function of the rods’ contact angle, but also governed by their static pre-compression represented by \( \delta_0 \).

Based on the aforementioned linear contact relationship, a discrete particle model can be established for a chain of cylinders in alternating contact angles (\( \alpha_1 \) and \( \alpha_2 \)), using point masses connected by springs (Fig. 3a). According to Eq. (2), the linearized contact stiffness \( \beta_1 \) and \( \beta_2 \) for the contact angles \( \alpha_1 \) and \( \alpha_2 \) can be expressed as:

\[
\beta_1 = \frac{3}{2} k_{cyl}(\alpha_1) \delta_0^{1/2} = \frac{3}{2} \left[ k_{cyl}(\alpha_1) \right]^{2/3} F_0^{1/3}
\]

\[
\beta_2 = \frac{3}{2} k_{cyl}(\alpha_2) \delta_0^{1/2} = \frac{3}{2} \left[ k_{cyl}(\alpha_2) \right]^{2/3} F_0^{1/3}
\]

where \( F_0 \) is the static pre-compression force. This confirms the tunable nature of the contact stiffness by manipulating \( F_0 \), which stems inherently from the nonlinearity of the contact mechanism.

Given the alternating spring connections, a unit cell of this model is composed of two particles as shown in Fig. 3a. The linearized equations of motion for the two elements in the unit cell are as follows:

\[
\begin{align*}
\dot{m_i} u_i &= \beta_1 (u_{i-1} - u_i) - \beta_2 (u_i - u_{i+1}) \\
\dot{m_i} v_i &= \beta_2 (u_i - v_i) - \beta_1 (v_i - u_{i+1})
\end{align*}
\]
To have a nontrivial solution for Eq. (6), the determinant of the matrix should be zero, and therefore, the system should satisfy the following relationship:

$$m^2 \omega^4 - 2(\beta_1 + \beta_2)\omega^2 + 2 \beta_1 \beta_2 (1 - \cos (kd)) = 0$$

(7)

This equation provides the information about the angular frequencies of transmitted linear elastic waves as a function of wavenumber $k$ as shown in Fig. 4a. This set of solutions defines the dispersion mechanism of linear elastic waves propagating in the phononic crystal chain. This dispersion relationship allows the transmission of elastic waves at specific frequency ranges (called pass-bands), while they block certain frequencies (called stop-bands or bandgap). The upper and lower frequency bands are sometimes referred to as optical and acoustic pass-bands, though our system does not have anything to do with optics.

We can analytically obtain the cutoff frequencies of these bands based on the dispersion relationship in Eq. (7). For example, when wavenumber $k$ is zero (i.e., infinitely long wavelength), we obtain two cutoff frequencies, $\omega_{\text{lower/AB}} = 0$ and $\omega_{\text{upper/AB}} = \sqrt{2(\beta_1 + \beta_2)/m}$, where subscripts AB and OB stand for acoustic and optical bands respectively. In the same way, the other two cutoff frequencies, $\omega_{\text{upper/OB}} = \sqrt{2\beta_1/m}$ and $\omega_{\text{lower/OB}} = \sqrt{2\beta_2/m}$ (when $\beta_2 \geq \beta_1$), are obtained when the wavenumber is equal to $\pi/d$ (corresponds to the shortest wavelength of 2$d$). It should be noted that these analytical dispersion curves are obtained under the assumption of an infinitely long chain of quasi-1D phononic crystals. In fact, we did not account for any boundary conditions in the aforementioned derivation processes.

To identify the frequency response of the “finite” chain, we need to model the boundary conditions imposed on the first and last particles. Mathematically, the equations of motion of the boundary particles in the given setup (Fig. 3a) are:

$$m\ddot{u}_1 = \frac{3}{2}k_{\text{act}}\delta_{0}^{1/2}(-u_1) - \beta_1(u_1 - v_1)$$

$$m\ddot{v}_N = \beta_1(u_N - v_N) - \frac{3}{2}k_{\text{act}}\delta_{0}^{1/2}(v_N)$$

(8)
Here $N$ is the total number of unit cells in the chain, and $k_{\text{act}}$ ($k_{\text{sen}}$) is the Hertzian contact stiffness between the first (last) particle and the actuator (sensor). By combining the equations of motion in Eqs. (4) and (8), we can construct the dynamic system in a state-space form as follows:

$$\begin{align*}
\dot{x} &= Ax + BF_1 \\
F_N &= Cx + DF_1
\end{align*}$$

where $x$ is the state vector and $A$, $B$, $C$, and $D$ are state, input, output, and transmission matrices [17]. $F_1$ is the input imposed on the system, which is the excitation force applied by the actuator. The output of the system $F_N$ is represented by the dynamic contact force transmitted to the sensor at the end of the chain. Given the experimental setup described in Section 2, $D$ should be a null matrix with a $[1 \times 1]$ size. The other matrices are:

$$x = \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_N \\ v_N \end{pmatrix}, \quad A = \begin{pmatrix} 0 & I \\ M^{-1}K & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1/m \end{pmatrix}, \quad C = \begin{pmatrix} 0 & \cdots & 0 & k_{\text{sen}} & 0 & \cdots & 0 \end{pmatrix}$$
where \( M \) and \( K \) sub-matrices are expressed as:

\[
M = \begin{bmatrix}
m & 0 & \cdots & 0 & 0 \\
0 & m & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & m & 0 \\
0 & 0 & \cdots & 0 & m \\
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
-k_{\text{act}} & k_{\alpha 1} & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & -k_{\alpha 1} & -k_{\alpha 2} & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & -k_{\alpha 2} & -k_{\alpha 1} & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & k_{\alpha 2} & -k_{\alpha 1} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & k_{\alpha 1} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & k_{\alpha 1} & -k_{\text{sen}} \\
\end{bmatrix}
\]

We obtain the transfer function of the system and calculate its transmission gain using MATLAB’s `bode` function. Note that this state-space approach does not require initial conditions of the excitations, since we are focused on steady-state solutions of the system in terms of transmission gains, instead of its transient solutions. The results will be discussed in Section 4. For the sake of simplicity, we assume that the two stiffness values \( (k_{\text{act}} \text{ and } k_{\text{sen}}) \) are the same as \( k_{\text{cyl}} \).

### 3.2. Finite element model

To consider the cylinder’s complex rotational dynamics under non-zero offsets, we use a commercial finite element program COMSOL. In this simulation, we systemically vary the center-of-mass offsets of aligned particles under the identical pre-compression of 25.0 N and the 90° contact angle. Instead of modeling the full scale of the chain, we take into account only a unit cell composed of two tightly adjoined cylinders and impose periodic conditions at the particle boundaries under the assumption of an infinite chain. This is not only to save the computational time, but also to obtain continuous dispersion curves (Fig. 4b) comparable to the analytical solutions as plotted in Fig. 4a.

Fig. 3b shows an example of the finite element model based on tetrahedral mesh configurations. In this model, we create a high density of meshes at the particle interface to capture accurate contact dynamics. For this, we first calculate the penetration depth \( \delta_0 \) (i.e., overlapping distance) of the two cylinders at the contact point based on the Hertzian contact theory in Section 3.1:

\[
\delta_0 = \frac{3(1-\nu^2)F_0}{2E\sqrt{R}} \quad \cdot 2^{3/2}
\]

In our finite element model, we shift two particles towards each other by \( \delta_0 \) and then merged them into a unit entity to mimic this Hertzian contact configuration. For the boundaries on the left and right hand side particles, two circular areas are defined which have radius \( (R_c) \) equivalent to the Hertzian contact area of two contacting cylinders under pre-compression. According to the Hertzian contact law, this radius can be calculated by \( R_c = \sqrt{R \delta_0} \) for a spherical contact area [15]. We impose the periodic boundary condition to the two circular areas at both ends of the model in the axial direction. All other surfaces are considered as non-constrained and free boundaries. We vary the wavenumber \( k \) and investigate the modal shapes of the unit cell at different frequencies, which ultimately result in smooth curves of dispersion relationship.

### 4. Results and discussion

#### 4.1. Contact angle variations

We begin with discussing the effect of contact angle variations in quasi-1D phononic crystals given zero center-of-mass offsets. Fig. 4b and c shows the numerical and experimental results of the phononic crystal’s frequency responses (i.e., transmitted gains) in terms of power spectral density (PSD) under alternating contact angles of \( \alpha_1=15° \) and \( \alpha_2=90° \). As predicted by the analytical dispersion curves in Fig. 4a, we verify the presence of the evident band gap between two pass-bands (located approximately between 8 and 11 kHz). The cutoff frequencies obtained from the numerical state-space approach (vertical blue lines in Fig. 4b) agree well with the analytical ones based on the dispersion relationship in an infinite chain (Fig. 4a). The minute discrepancies between the analytical and numerical results are attributed to the finite and discrete nature of the phononic crystal tested in this study.

In Fig. 4b, the spikes in the frequency spectrum represent the resonant modes of the system. For example, Fig. 5a and b depict the modal shapes of the first modes in the acoustic and optical bands (i.e., first resonant modes in the lower and upper pass-bands) based on the eigen-analysis of the state-space matrices as discussed in Section 3.1. The first acoustic
Fig. 5. Modal shapes of the cylindrical chain with alternating contact angles ($\alpha_1 = 15^\circ$, $\alpha_2 = 90^\circ$) and 25 N pre-compression. (a) 1st acoustic modal shape. (b) 1st optical modal shape. Bottom figures in each panel illustrate the relative positions of unit cells composed of two masses (marked in circles and squares).

Fig. 6. Evolution of frequency band structures as a function of $\alpha_2$ ($\alpha_1$ is kept constant at 90°). (a) Numerical results based on the state-space approach. The gray and white areas represent allowable and forbidden zones. The red dots denote cutoff frequencies obtained from numerical simulations, and these dots are interpolated to obtain the boundaries of the gray zone. (b) Experimental results for $\alpha_2 = [15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ]$. The color intensity represents the transmission gain in terms of PSD (see the colormap in dB on the right). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
mode shows in-phase oscillations of all particles relative to their equilibrium positions, whereas the first optical mode exhibits out-of-phase vibrations of unit cells. To ease visualization, the bottom figures of each panel in Fig. 5 illustrate the relative positions of the masses in the chain, in which two particles of a unit cell (marked in circles and squares) are under in-phase and out-of-phase oscillations. From these plots, it is evident that the spatial wavelength of the acoustic mode is longer than that of the optical mode, which is in agreement with the analytical dispersion curve in Fig. 4a.

The experimental results in Fig. 4c corroborate the numerical and analytical results qualitatively. We evidently observe the formation of the optical pass-band, though its cutoff frequencies are noticeably down-shifted and its transmission gains are reduced compared to those of the acoustic band. This is probably due to the enhanced degree of friction (among particles as well as against the supporting ladders) in the optical vibration modes, which are characterized by the relatively high frequency oscillations relative to the acoustic modes.

The evolution of frequency band structures is illustrated numerically and experimentally in Fig. 6a and b respectively as a function of alignment angles. Herein, we vary the contact angle $\alpha_2$ in six steps from 15° to 90°, while $\alpha_1$ is kept identical to 90°. When both contact angles are equal at 90°, the phononic crystal acts like a monoatomic system with the uniform mass and stiffness distribution. In this case, there is a wide passband formed from 0 to 11 kHz according to the numerical simulations as shown in Fig. 6a. By introducing the smaller $\alpha_2$, the system progresses more towards a diatomic configuration due to the discrepancies of the contact stiffness values within a two-particle unit cell. As a result, an intermediate bandgap starts to appear in the mid-range frequency as shown in Fig. 6a. As $\alpha_2$ decreases further, the associated contact stiffness increases, and this in turn shifts the optical band to the higher frequency range. It should be noted that the cutoff frequency of the acoustic band remains constant regardless of the $\alpha_2$ variations. This is because the upper cutoff frequency of the acoustic band is characterized solely by the stiffness of the other, undisturbed contact (i.e., $\omega_{\text{upper}}_{AB} = \sqrt{2\beta_1/m}$ as derived in Section 3.1). The tunable characteristics of quasi-1D cylindrical phononic crystals is successfully validated by experimental tests as shown in Fig. 6b. The noticeable discrepancies between the numerical and experimental results are probably due to the imperfect test setup, which is susceptible to misalignments of particles and frictional effects.

Fig. 7. Dispersion curves for three different sets of offsets ($[e_1, e_2]= [7.0, 0.0]$, [7.0, 3.5], and [7.0, 7.0] mm for (a), (b), and (c)). Insets show the vibrational mode shapes of a unit cell in various branches of dispersion curves. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)
4.2. Center-of-mass offset variations

Now we investigate the effect of center-of-mass offsets on the vibrational modes of quasi-1D phononic crystals. Fig. 7 shows the dispersive behavior of three different offset configurations: $[e_1, e_2] = [7.0, 0.0], [7.0, 3.5]$, and $[7.0, 7.0]$ mm based on finite element simulations. The movements of the unit cell under the wavenumber of $k = \pi / 2d$ (red dots in Fig. 7) are illustrated in the insets, where highlighted colors and arrows represent maximum displacements and displacement directions, respectively. For all three cases, we observe complicated, multiple pass-bands in sharp contrast to the zero-offset.

![Fig. 8. Evolution of frequency band structures in quasi-1D phononic crystals as a function of the cylinders’ offsets based on numerical (left panel) and experimental (right) approaches. (a–b) $e_1 = 0.0$ mm. (c–d) $e_1 = 3.5$ mm. (e–f) $e_1 = 7.0$ mm. In numerical simulations, $e_2$ is varied from 0.0 mm to 7.0 mm with an increment of 1.0 mm, while in experiments, the increment is 0.5 mm. The color intensity in the experimental surface plots implies the transmission gain in terms of PSD [dB] as shown in the colormap on the right. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)](image-url)
The dispersion curve becomes wider as the offset increases. We also observe that when the two offsets (compare the insets and dispersion curves corresponding to the fifth mode in Fig. 7a, b, and c). It is notable that the width of the pass-bands, similar to the formation of the acoustic/optical pass-bands as discussed in Section 4.1. This was also observed for ineffective shear modes around 12 kHz slowly becomes effective rocking modes as we impose larger offsets (compare the insets and dispersion curves corresponding to the fifth mode in Fig. 7a, b, and c). It is notable that the width of the dispersion curve becomes wider as the offset increases. We also observe that when the two offsets $e_1$ and $e_2$ are the same (Fig. 7c), each pair of the lower and upper pass-bands meet together at $k = \pi / d$. This reduces the complicated band structure into two effective pass-bands, similar to the formation of the acoustic/optical pass-bands as discussed in Section 4.1. This was also observed in our previous study on the symmetric phononic crystal [14].

To observe how the frequency band structures evolve in relation to offsets, we gradually vary $e_2$ from 0.0 mm to 7.0 mm, while we fix $e_1$, the offset of the other ladder, to a constant value. The right and left panels of Fig. 8 show the numerical and experimental results under the representative values of fixed $e_1$ at 0.0, 3.5, and 7.0 mm. The red dots in the numerical plots correspond to cutoff frequencies obtained from the finite element simulations, and we interpolate these discrete points to construct smooth zones of pass-bands (gray areas in Fig. 8). First, the numerical and experimental results of band structure progression for $e_1 = 0.0$ mm are presented in Fig. 8a and b. We observe that given a moderate offset, the phononic crystal exhibits only a single pass-band, showing no divergence into multiple bands. However, at $e_2 = 3.0$ mm, we see the emergence of a band-gap around 9 to 10 kHz. If the offset increases further, we witness the formation of an additional band-gap, which starts around 11 kHz then widens as $e_2$ increases. The experimental results successfully validate this branching behavior. The blunt colors in the bifurcated pass-bands imply a relatively low transmission gain, which is probably due to the high friction and energy dissipation experienced by the particles under the violent rocking motions.

The middle and bottom panels in Fig. 8 show the evolution of band structures for $e_1 = 3.5$ and 7.0 mm. We clearly observe the formation of the upper bands corresponding to the rocking motions of the particles. However, the middle bands predicted by the numerical simulations are faint and barely visible in Fig. 8d. As explained above, this is because that these middle bands are created by the shear motions of the particles, and thus, they are not easily measured by the force sensor. The band structure for $e_1 = 7.0$ mm also shows the creation of multiple pass-bands due to the combined behavior of translational, rotational, and rocking motions. The experimental results corroborate the numerical simulations qualitatively. Overall, we observe that the center-of-mass offsets trigger rich dynamics of band structures’ branching behavior, despite the simple architectures of homogeneous phononic crystals. We find this mechanism results from the evolution of vibration modes and bandgaps, which can be further facilitated by introducing asymmetric offsets.

5. Conclusion

In this paper, we presented the numerical and experimental work to investigate the tunability of quasi-1D phononic crystals composed of chains of cylindrical particles. We showed that by manipulating two simple geometrical parameters: contact angles and center-of-mass offsets between neighboring particles, we can controllably tune the frequency responses of phononic crystals in terms of cutoff frequency locations and the number of pass-bands. We first demonstrated the formation of a single bandgap in a monoatomic chain by introducing alternating contact angles. By simply manipulating the contact angles, we showed that the bandgap can be widened due to the discrepancies between the two contact angles. We also found that the geometrical offsets of the aligned phononic crystals can create complex frequency band structures due to the coupling of cylinders’ translational, rotational and shear motions. We analyzed the behavior of the generated modes and elucidated the evolution of such modes, which can be converted from ineffective modes into visible, effective modes and vice versa. The analytical and numerical approaches based on discrete particle and finite element methods successfully predicted the versatile branching mechanisms of frequency band structures in symmetric and asymmetric phononic crystals. The simulation results agreed well with the experimental outcome. While this study is restricted to the quasi-1D behavior of phononic crystals, the analytical and numerical tools developed in this study are expected to be useful for multidimensional phononic crystals. The findings in this study can potentially contribute to the development of meta-materials for tunable filtering of mechanical waves in an efficient and controllable manner.

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